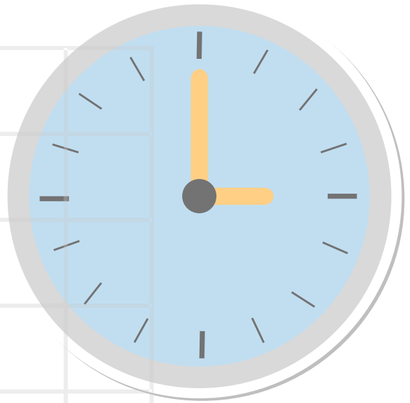
SECOND ORDER DIFFERENTIATION

The derivative of a derivative is called the second order derivative

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) or \ \mathbf{f''(x)}$$

Find second order differentiation f''



$a) y = 10x^4 - 5x^3 + 3x - 7$

Solution:

First derivative: $\frac{dy}{dx} = 40x^3 - 15x^3 - 3$

second derivative: $\frac{d^2y}{dx^2} = 120x^3 - 45x^2$

$b) y = 5sin6x$

Solution:

First derivative: $\frac{dy}{dx} = 30cos6x$

second derivative: $\frac{d^2y}{dx^2} = -180sin6x$

$c) y = e^{x^2}$

Solution::

First derivative: $\frac{dy}{dx} = 2xe^{x^2}$

second derivative:

$u = 2x$ $\qquad$ $v = e^{x^2}$

$\frac{du}{dx} = 2$ $\qquad$ $\frac{dv}{dx} = 2xe^{x^2}$

$$\frac{d^2y}{dx^2} = 2x\left(2xe^{x^2}\right) + 2e^{x^2}$$

$$\frac{d^2y}{dx^2} = 4x^2e^{x^2} + 2e^{x^2} = 2e^{x^2}\left(2x^2 + 1\right)$$

$d) y = (x+3)^3$

Solution:

First derivative: $\frac{dy}{dx} = 3(x+3)^{3-1}\frac{d}{dx}(x+3)$

$= 3(x+3)^2(1)$

$= 3(x+3)^2$

second derivative: $\frac{d^2y}{dx^2} = (2)(3)(x+3)^{2-1}\frac{d}{dx}(x+3)$

$= 6(x+3)^1(1)$

$= 6(x+3)$

$e) y = (2x^2-2)(x^2+3)$

Solution:

First derivative: $u = 2x^2 - 2 \quad v = x^2 + 3$

$\frac{du}{dx} = 4x \qquad \frac{dv}{dx} = 2x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx} = (2x^2-2)(2x) + (x^2+3)(4x)$

$= 4x^3 - 4x + 4x^3 + 12x$

$= 8x^3 + 8x$

second derivative: $\frac{d^2y}{dx^2} = (3)(8x^{3-1}) + 8(1)$

$= 24x^2 + 8$

$$f)y = \frac{x-2}{x+3}$$

Solution:

First derivative: $u = x - 2$ $\qquad$ $v = x + 3$

$$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(x+3)(1) - (x-2)(1)}{(x+3)^2}$$

$$\frac{dy}{dx} = \frac{x+3-x+2}{(x+3)^2}$$

$$= \frac{5}{(x+3)^2}$$

second derivative: $\frac{d^2y}{dx} = 5(x+3)^{-2}$

$$= (-2)(5)(x+3)^{-2-1}$$

$$= -10(x+3)^{-3}$$

$$g)y = 3ln(5x+7)$$

Solution:

First derivative: $\frac{dy}{dx} = 2ln(5x+7)\frac{d}{dx}(5x+7)$

$$= 2.\frac{1}{5x+7}.5$$

$$= \frac{10}{5x+7}$$

second derivative: $\frac{d^2y}{dx^2} = 10(5x+7)^{-1}$

$$= -1(10)(5x+7)^{-1-1}$$

$$= -10(5x+7)^{-2}$$

LET`S TRY THIS !!!

FIND THE HIGHER DERIVATIVE OF $\frac{d^2y}{dx^2}$

$a) y = 5x^3$

$b) y = (x^3 + 2x)^2$

$c) y = 2x^3 + 5x^2 - 7x + 6$

$d) y = ln\sqrt{5x + 3}$

$e) y = e^{9x+2}$

$f) y = 2sin6x$

ALL the BEST!!!

Answer:

$a) \frac{dy}{dx} = 30x$

$b) \frac{dy}{dx} = 30x^4 + 48x^3 + 8$

$c) \frac{dy}{dx} = 12x + 10$

$d) \frac{dy}{dx} = -\frac{25}{2}(5x + 3)^{-2}$

$e) \frac{dy}{dx} = 81e^{9x+2}$

$f) \frac{dy}{dx} = -72sin6x$