DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

THE BASIC DERIVATIVES:

- $\frac{d}{dx}(sinx) = cosx$
- $\frac{d}{dx}(secx) = secxtanx$
- $\frac{d}{dx}(cosx) = -sinx$
- $\frac{d}{dx}(cosecx) = -cosecxcotx$
- $\frac{d}{dx}(tanx) = sec^2x$
- $\frac{d}{dx}(cotx) = -cosec^2x$

- $y = sin\,(ax) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}sin\,(ax)\frac{d}{dx}ax$
  $= cos\,(ax).\,(a)$
  $= acos\,(ax)$

- $y = cos\,(ax) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}cos\,(ax)\frac{d}{dx}ax$
  $= -sin\,(ax).\,(a)$
  $= -asin\,(ax)$

- $y = tan\,(ax) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}tan\,(ax)\frac{d}{dx}ax$
  $= sec^2\,(ax).\,(a)$
  $= asec^2\,(ax)$

- $y = sin^n\,(ax)$
  $\Rightarrow \frac{dy}{dx} = \frac{d}{dx}sin^n\,(ax)\frac{d}{dx}sin\,(ax)\frac{d}{dx}ax$
  $= nsin^{n-1}\,(ax)cos\,(ax).\,(a)$
  $= ansin^{n-1}\,(ax)cos\,(ax)$

- $y = cos^n\,(ax)$
  $\Rightarrow \frac{dy}{dx} = \frac{d}{dx}cos^n\,(ax)\frac{d}{dx}cos\,(ax)\frac{d}{dx}ax$
  $= ncos^{n-1}\,(ax).-sin\,(ax).\,(a)$
  $= -ancos^{n-1}\,(ax)sin\,(ax)$

- $y = tan^n\,(ax)$
  $\Rightarrow \frac{dy}{dx} = \frac{d}{dx}tan^n\,(ax)\frac{d}{dx}tan\,(ax)\frac{d}{dx}ax$
  $= ntan^{n-1}\,(ax).\,sec^2\,(ax).\,(a)$
  $= antan^{n-1}\,(ax)sec^2\,(ax)$

Example

If $y = 3tanx$, then

$$= 3\frac{d}{dx}tanx$$

$$\frac{dy}{dx} = 3sec^2x$$

Example

If $y = 5cos4x$, then

$$= 5\frac{d}{dx}cos\,(4x)\frac{d}{dx}(4x)$$

$$= 5.-sin\,(4x).\,(4)$$

$$\frac{dy}{dx} = -20sin4x$$

Example

If $y = sin^3\,(3x^2)$, then

$$= \frac{d}{dx}sin^3\,(3x^2)\frac{d}{dx}sin\,(3x^2)\frac{d}{dx}(3x^2)$$

$$= 3sin^{3-1}\,(3x^2).\,cos\,(3x^2).\,(6x)$$

$$\frac{dy}{dx} = 18xsin^2\,(3x^2).\,cos\,(3x^2)$$

DIFFERENTIATION OF LOGARITHMIC FUNCTIONS

THE BASIC DERIVATIVES:

Given $y = lnx$

then

$$\frac{d}{dx}(lnx) = \frac{1}{x} \quad \longrightarrow \quad x > 0$$

Given $y = ln\,(ax+b)$

then

$$\frac{d}{dx}ln\,(ax+b) = \frac{d}{dx}ln\,(ax+b)\frac{d}{dx}(ax+b)$$

$$= \frac{1}{(ax+b)}(a)$$

$$= \frac{a}{(ax+b)}$$

Example

Differentiate

a) $y = ln2x$

$$= \frac{d}{dx}ln2x\frac{d}{dx}(2x)$$

$$= \frac{1}{2x}(2)$$

$$\frac{dy}{dx} = \frac{1}{x}$$

b) $y = ln\,(2x^3+3)$

$$= \frac{d}{dx}ln\,(2x^3+3)\frac{d}{dx}(2x^3+3)$$

$$= \frac{1}{(2x^3+3)}(6x^2)$$

$$\frac{dy}{dx} = \frac{6x^2}{2x^3+3}$$

THE LOGARITHMIC RULES:

a) $ln\,(xy) = lnx + lny$

b) $ln\frac{x}{y} = lnx - lny$

c) $lnx^n = nlnx$

d) $lnx^n = ln\,(xx)$

$$= lnx + lnx$$

$$= 2lnx$$

e) $(lnx)^n = n(lnx)^{n-1}\frac{d}{dx}(lnx)\frac{d}{dx}(x)$

Example

Differentiate

$$y = \frac{2}{x}$$

$$= ln2 - lnx$$

$$= \frac{d}{dx}(ln2) - \frac{d}{dx}(lnx)$$

$$= 0 - \frac{1}{x}$$

$$\frac{dy}{dx} = -\frac{1}{x}$$

DIFFERENTIATION OF EXPONENTIAL FUNCTIONS

THE BASIC DERIVATIVES:

Given

- $y = e^x \Rightarrow \frac{dy}{dx} = \frac{d}{dx}e^x = e^x$
- $y = e^{ax} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}e^{ax}\frac{d}{dx}(ax) = ae^{ax}$
- $y = e^{ax+b}$

  $\Rightarrow \frac{dy}{dx} = \frac{d}{dx}e^{ax+b}\frac{d}{dx}(ax+b) = ae^{ax+b}$

Example

▷ Differentiate

a) $y = e^{2x}$

$$= \frac{d}{dx}e^{2x}\frac{d}{dx}(2x)$$

$$= e^x\,(2)$$

$$\frac{dy}{dx} = 2e^x$$

b) $y = 2e^{5x+7}$

$$= 2\frac{d}{dx}e^{5x+7}\frac{d}{dx}(5x+7)$$

$$= 2e^{5x+7}\,(5)$$

$$\frac{dy}{dx} = 10e^{5x+7}$$

THE EXPONENTIAL RULES:

a) $e^{x+y} = e^x e^y$

b) $e^{x-y} = \frac{e^x}{e^y}$

c) $e^{-x} = \frac{1}{e^x}$

d) $(e^m)^n = e^{m \times n} = e^{mn}$

e) $e^0 = 1$

Example

▷ Differentiate

$$y = e^x\left(e^{3x} - e^{5x}\right)$$

$$= e^x e^{3x} - e^x e^{5x}$$

$$= e^{x+3x} - e^{x+5x}$$

$$y = e^{4x} - e^{6x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{4x} + e^{6x}\right)$$

$$= \frac{d}{dx}e^{4x}\frac{d}{dx}(4x) - \frac{d}{dx}e^{6x}\frac{d}{dx}(6x)$$

$$= e^{4x}\,(4) - e^{6x}\,(6)$$

$$\frac{dy}{dx} = 4e^{4x} - 6e^{6x}$$

LET'S DO EXERCISE

Differentiation of Trigonometric Functions

1	$y = sin5x$	Answer:	$\frac{dy}{dx} = 5cos5x$
2	$y = 5cos4x$	Answer:	$\frac{dy}{dx} = -20sin4x$
3	$y = 2tan\,(2x-3)$	Answer:	$\frac{dy}{dx} = 4sec^2\,(2x-3)$
4	$y = 2sin\left(3x^2-5x\right)$	Answer:	$\frac{dy}{dx} = 2\,(6x-5)cos\left(3x^2-5x\right)$
5	$y = cos^3x$	Answer:	$\frac{dy}{dx} = -3cos^2xsinx$
6	$y = tan^2 3x$	Answer:	$\frac{dy}{dx} = 6tan3xsec^2 3x$
7	$y = 2cos^4\,(3x-4)$	Answer:	$\frac{dy}{dx} = -24cos^3\,(3x-4)sin\,(3x-4)$

Differentiation of Logarithmic Functions

1	$y = \frac{1}{3}lnx$	Answer:	$\frac{dy}{dx} = \frac{1}{3x}$
2	$y = ln5x$	Answer:	$\frac{dy}{dx} = \frac{1}{x}$
3	$y = ln\left(3+x^2\right)$	Answer:	$\frac{dy}{dx} = \frac{2x}{3+x^2}$
4	$y = ln\left(x^3+3x-4\right)$	Answer:	$\frac{dy}{dx} = \frac{3x^2+3}{x^3+3x-4}$
5	$y = 5ln\,(10x+3)$	Answer:	$\frac{dy}{dx} = \frac{50}{10x+3}$

Differentiation of Logarithmic Functions (Use logarithmic rules)

1	$y = ln\left(x^5\right)$	Answer:	$\frac{dy}{dx} = \frac{5}{x}$
2	$y = ln(3x+2)^5$	Answer:	$\frac{dy}{dx} = \frac{15}{3x+2}$
3	$y = ln\,(x+1)\,(2x-3)$	Answer:	$\frac{dy}{dx} = \frac{1}{x+1} + \frac{2}{2x-3}$
4	$y = ln\left(\frac{5x^3}{7}\right)$	Answer:	$\frac{dy}{dx} = \frac{3}{x}$
5	$y = ln\left(\frac{3x}{x+1}\right)$	Answer:	$\frac{dy}{dx} = \frac{1}{x^2+x}$

LET'S DO EXERCISE

Differentiation of Exponential Functions

1	$y = 5e^{6x}$	Answer: $\frac{dy}{dx} = 30e^{6x}$
2	$y = e^{3x+2}$	Answer: $\frac{dy}{dx} = 3e^{3x+2}$
3	$y = e^{-4x+1}$	Answer: $\frac{dy}{dx} = -4e^{-4x+1}$
4	$y = 4e^{x^2}$	Answer: $\frac{dy}{dx} = 8xe^{x^2}$
5	$y = e^{-2x^3}$	Answer: $\frac{dy}{dx} = -6x^2e^{-2x^3}$
6	$y = 2e^{-2x} + 3e^{3x}$	Answer: $\frac{dy}{dx} = -4e^{-2x} + 9e^{3x}$
7	$y = \frac{e^{4x}}{4} + \frac{e^{5x}}{5}$	Answer: $\frac{dy}{dx} = e^{4x} + e^{5x}$

Differentiation of Exponential Functions (Use exponential rules)

1	$y = \frac{1}{e^{3x}}$	Answer: $\frac{dy}{dx} = -\frac{3}{e^{3x}}$
2	$y = e^{2x} - \frac{1}{e^{3x}}$	Answer: $\frac{dy}{dx} = 2e^{2x} + 5e^{-5x}$
3	$y = e^{3x}\left(1 - 5e^{-4x}\right)$	Answer: $\frac{dy}{dx} = 3e^{3x} + 5e^{-x}$
4	$y = \frac{e^{7x}}{e^{x}}$	Answer: $\frac{dy}{dx} = 6e^{6x}$
5	$y = \frac{3 - e^{5x}}{e^{3x}}$	Answer: $\frac{dy}{dx} = -9e^{-3x} - 2e^{2x}$