3.1 PROPOSITIONAL LOGIC

3.1.1 COMPOUND PROPOSITION

1) Purpose of Proposition Logic

Proposition is a *declarative sentence* that is either *true or false* but not both. A *primary* (*simple*) *statement* is a statement that can be represented by a variable, like P, Q, R, S or *p*, *q*, *r*, *s*. The *truthfulness or falsity* of a statement is called its *truth value*. A true statement has truth value T or 1, a false statement has truth value F or 0. Sentences in the form of question, command or variables are not propositions.

Logic is the study of correct reasoning. Use of logic include:

- In mathematics:
  - To prove theorems

- In computer science:
  - To prove that programs do what they are supposed to do
  - Programming language
  - Design of computer circuits
  - The construction of computer programs
  - The verification of the correctness of programs
  - Design of computing machines
  - System specifications
  - Artificial intelligence

All the following declarative sentences are **propositions**.

a. Kuala Lumpur is the capital of Malaysia.
b. There is 25 hours in one day.
c. 1+1 = 2
d. 2+2 = 3

All the following declarative sentences are **not propositions**.

a. What time is it?
b. Read this carefully.
c. $x + 1 = 2$
d. $x + y = z$

Exercise 1:

Which of the following are propositions? For a proposition, find its truth value.

No.	Sentences	Proposition / Not	Truth Value
1	Where do you come from?		
2	2 + 3 = 6		
3	What a beautiful flower!		
4	Is Mr. Lau tall?		
5	Take two aspirins.		
6	$13 - x = 8$, where $x = 5$		
7	2 < 4		
8	$x + 5 = 10$, where $x = 3$		
9	x is an even number.		
10	$x - 1$		
11	7 is an odd number.		
12	The earth is flat.		
13	Please sit down.		
14	Jitra is a capital city of Perlis.		
15	Answer this question.		

2) Logical Operators

A propositional formula is constructed from simple propositions or propositional variables such as P and Q, using logical connectives or logical operators such as NOT, AND, OR, IF AND ONLY IF or IMPLIES.

a. Negation

Symbol: $\sim$ or $\neg$

The negation of a statement P is **not** P. Turn a true proposition into false or a false proposition into true.

Exercise 2:

Give the negation of the following statements.
1. *p*: It is cold.
2. *q:* $2 + 3 > 1$
3. *r*: I have Brown hair

b. Conjunction

Symbol: $\wedge$

Read as *"p* **and** *q" or "p* **but** *q".* The proposition is TRUE only when *p* and *q* are both true.

Exercise 3:

1. Find the conjunction of the propositions *p* and *q* where
   p: Today is Monday
   q: It is raining today.

2. Form the conjunction of *p* and *q* for each of the following:
   p: It is snowing
   q: I am cold

c. Disjunction

Symbol: $\vee$

Read as *"p* **or** *q".* The proposition is FALSE only when *p* and *q* are both false.

Exercise 4:

1. Find the disjunction of the propositions *p* and *q* where
   p: Today is Monday
   q: It is raining today.

2. Find the disjunction of the propositions *p* and *q* where
   p: My car has a bad engine
   q: My car has a bad carburetor

3. Form the disjunction of *p* and *q* for each of the following:
   a. *p*: 2 is positive integer
      q: $2 + 3 = 6$

   b. *p*: The computer program has a bug
      q: The input is erroneous

   c. *p*: I drove to work
      q: I took the train to work

   d. *p*: $2 > 1$
      q: $-1 < -2$

d. Conditional

Symbol: $\rightarrow$

Read as "**if** p **then** q", where p is called hypothesis and q is called the conclusion. The proposition is TRUE only when p and q are both true and p is false (no matter what truth value q has).

$p \rightarrow q$ also can be read as:

- ***If*** p ***then*** q
- ***If*** p, q
- p ***is sufficient for*** q
- p ***only if*** q
- p ***implies*** q
- ***a necessary condition for*** p ***is*** q
- p ***is*** q

- q ***if*** p
- q ***when*** p
- q ***whenever*** p
- q ***is necessary for*** p
- q ***follows from*** p
- q ***unless*** $\sim p$
- ***a sufficient condition for*** q ***is*** p

There are some related implications that can be formed from $p \rightarrow q$ which are converse, contrapositive and inverse.

i. **Converse**
$q \rightarrow p$

ii. **Contrapositive**
$\sim q \rightarrow \sim p$

iii. **Inverse**
$\sim p \rightarrow \sim q$

Exercise 5:

1. Find the implication of the propositions p and q where
   a. p: Maria learns discrete mathematics
      q: Maria will find a good job

   b. p: It is raining
      q: The home team wins

   c. p: I am hungry
      q: I will eat

   d. p: $3 + 5 = 8$
      q: $8 - 3 = 5$

2. Give the converse, contrapositive and inverse of the implication:
   a. "If today is holiday then class is cancelled."

   b. "I need an umbrella whenever it is raining."

   c. "If it is sunny tomorrow, I will go for a walk in the woods."

e. Biconditional

Symbol: $\leftrightarrow$

Read as "*p* **if and only if** *q*". The proposition is TRUE when *p* and *q* have the same truth values.

Exercise 6:

1. Find the biconditional statement of the propositions *p* and *q* where
   a. *p*: Maria learns discrete mathematics
      q: Maria is an IT student

   b. *p*: You can take the flight
      q: You buy a ticket

   c. *p*: I am happy
      q: I am healthy

   d. *p*: 3 + 5 = 8
      q: 8 – 3 = 5

3) Compound Proposition

A compound proposition is a proposition that involves the assembly of multiple statements.

Exercise 7:

1. Let

   P: I am rich;

   Q: I am happy.

   Write the following compound statements in symbolic forms.

   a. I am rich and happy.
   b. I am poor but happy.
   c. I am not rich and not happy.
   d. If I am happy then I am not poor.
   e. It is not true that if I am poor, then I am not happy.
   f. I am rich is the necessary condition for me to be happy.

2. Let p and q be the propositions

   p: It is below freezing

   q: It is snowing

   Write these propositions using p and q and logical connectives.

   a. It is below freezing and snowing.
   b. It is below freezing but not snowing.
   c. It is not below freezing and it is not snowing.
   d. It is either snowing or below freezing (or both).
   e. If it is below freezing, it is also snowing.
   f. It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
   g. That it is below freezing is necessary and sufficient for it to be snowing.

3. Let p and q be the propositions

   p: You drive over 65 miles per hour

   q: You get a speeding ticket

   Write these propositions using p and q and logical connectives.

   a. You do not drive over 65 miles per hour.
   b. You drive over 65 miles per hour, but you do not get a speeding ticket.
   c. You will get a speeding ticket if you drive over 65 miles per hour.
   d. If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
   e. Driving over 65 miles per hour is sufficient for getting a speeding ticket.

3.1.2 TRUTH TABLE OF COMPOUND PROPOSITIONS

1) Output

A truth table is a way of organizing information to list out all possible scenarios.

AND

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

OR

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

IF THEN

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

IF AND ONLY IF

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

EXCLUSIVE OR

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

NOT

p	$\sim p$
0	1
1	0

Precedence of Logical Operators

1. ( )
2. $\sim$
3. $\wedge$
4. $\vee$
5. $\rightarrow$
6. $\leftrightarrow$

Tautology, Contingency and Contradiction

If the output of the truth table is **1**,
It is **Tautology**.

If the output of the truth table is **0 and 1**,
It is **Contingency**.

If the output of the truth table is **0**,
It is **Contradiction**.

a. Tautology

Compound statement (premise and conclusion) that is *always true* which normally shown by using truth table.

b. Contradiction

The opposite of a tautology is a contradiction, which is *always false*.

c. Contingency

A proposition that is neither a tautology nor a contradiction.

Exercise 8:

1. Construct the truth table of the compound proposition $(p \lor \sim q) \to (p \land q)$ and determine whether it is tautology, contradiction or contingency.

2. Construct the truth table of the compound proposition $[(p \to q) \land \sim q] \to (\sim p)$ and determine whether it is tautology, contradiction or contingency.

3. Construct the truth table of the compound proposition $(p \land \sim q) \to \sim r$ and determine whether it is tautology, contradiction or contingency.

4. Construct the truth table of the following compound proposition and determine whether it is tautology, contradiction or contingency.

a. $\sim p \vee q$

b. $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$

c. $(p \leftrightarrow q) \wedge (q \vee \sim p)$

d. $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

e. $(p \vee q) \leftrightarrow (q \vee p)$

6. Give the truth tables for

a. $(p \rightarrow r) \leftrightarrow (q \wedge \sim p)$

b. $\sim(p \vee q) \rightarrow (r \vee \sim q)$

c. $(p \rightarrow q) \vee (\sim p \rightarrow r)$

2) Valid Arguments in Propositional Logic

An argument is a sequence of statements. To say that an argument form is valid means that no matter what particular statements are substituted for the statement variables in its premises if the resulting premises are all true then the conclusion is also true. Each valid argument corresponds to an implication that is a **tautology**.

premises { Antecedent 1
Antecedent 2…

conclusion { ∴ consequence

Corresponding Tautology: [(antecedent…1) $\wedge$ (antecedent 2) $\wedge$ …] $\rightarrow$ consequence.

Therefore an argument $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is a valid argument $\leftrightarrow$ $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology.

To show the validity of compound propositions, construct a truth table for the premises and conclusion.

premises: $p \rightarrow q$, p — conclusion: q

p	q	$p \rightarrow q$	p	$[(p \rightarrow q) \wedge p]$	q	$[(p \rightarrow q) \wedge p] \rightarrow q$
0	0	1	0	0	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	1
1	1	1	1	1	1	1

Tautology
∴ valid

Exercise 9:

1. Determine whether these arguments are valid or not.

   a. John likes apple pies. Therefore, John likes apple pies or ice cream.

   b. Mary likes chocolate and ice cream. Therefore, Mary likes chocolate.

   c. If it snows, then the roads are closed; it snows. Therefore, the roads are closed.

2. Determine whether these arguments are valid or not.

   a. Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

b. Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

c. If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

d. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

e. If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

3. Use truth table to show that the hypotheses:

   Randy works hard,
   If Randy works hard then he is a dull boy,
   If Randy is a dull boy, then he will not get a job.

   Imply the conclusion "Randy will not get a job"

4. Show that the hypotheses

   If you send me an email message, then I will finish writing the program.
   If you do not send me an email message, then I will go to sleep early.

   Lead to the conclusion "If I do not finish writing the program, then I will go to sleep early"

5. Show that the hypotheses as below:

If this entry is the last, then the table is full.
If the table is full, it contains a marker.
The table contains no marker.

Lead to the conclusion “The entry is not the last”

6. Show that the hypotheses as below:

It is not sunny this afternoon and it is colder than yesterday,
We will go swimming only if it is sunny,

Lead to the conclusion “We will not go swimming”

3) Logical Equivalence

Symbol: $\equiv$ **or** $\Leftrightarrow$

- Compound propositions that have the same truth values in all possible cases are called **logically equivalent**.
- The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology.
- The notation $p \Leftrightarrow q$ / $p \equiv q$ denotes that p and q are logically equivalent
- Truth table is use to determine whether two compound propositions are equivalent.

Exercise 10:

1. Show that $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent.

2. Show that $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$.

3. Show that the propositions $\sim p \vee q$ and $p \rightarrow q$ are logically equivalent.

4. Show that $p \rightarrow q$ is logically equivalent to $\sim q \rightarrow \sim p$.

5. Show that $\sim(p \wedge \sim q)$ and $\sim p \vee q$ are logically equivalent.

6. Show that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically equivalent.

3.1.3 PROPOSITION LOGIC IN ENGLISH

1) Write a well-formed proposition logic in English sentences

Exercise 11:

1. For each of the symbolic expression, write the corresponding (compound) statement base on the given primary statements:
   P: Men are immortal.
   Q: Men are safe from tragedy.
   R: Men are created by God.

   a. $P \rightarrow Q$

   b. $P \rightarrow (Q \wedge R)$

   c. $Q \rightarrow \sim R$

   d. $\sim P \rightarrow (\sim Q \vee \sim R)$

   e. $(Q \wedge R) \leftrightarrow P$

2. Let p, q and r be the propositions
   p: You have the flu
   q: You miss the final examination
   r: You pass the course
   Express each of these compound propositions as an English sentence.

   a. $p \rightarrow q$

   b. $p \vee q \vee r$

   c. $\sim q \leftrightarrow r$

   d. $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$

   e. $q \rightarrow \sim r$

   f. $(p \wedge q) \vee (\sim q \wedge r)$

3.2 PREDICATE LOGIC

3.2.1 COMPOUND STATEMENT IN PREDICATE LOGIC

1) Definition of Predicates

The propositional logic is not powerful enough to represent all types of assertions that are used in computer science and mathematics, or to express certain types of relationship between propositions such as equivalence.

To cope with deficiencies of propositional logic, we introduce two new features: **predicates** and **quantifiers**.

> A **predicate** is a verb phrase template that describes a property of objects, or a relationship among objects represented by the variables.

2) Expression of Predicate in Statement

For example, the sentences

"The *car*, Karim is driving is blue",
"The *sky* is blue",
"The *cover* of this book is blue"

"is blue" is a predicate!

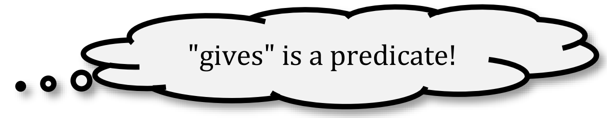
come from the template "is blue" by placing an appropriate noun/noun phrase in front of it. The phrase "is blue" is a predicate and it describes the property of being blue. Predicates often give a name. For example, any of "is_blue", "Blue" or "B" can be used to represent the predicate "is blue" among others.

If we adopt B as the name for the predicate "is_blue", sentences that assert an object is blue can be represented as "B(x)", where x represents an arbitrary object.

> B(x) reads as "x is blue".

Similarly, the sentences

"*Ali* <u>gives</u> the *book* to *Mariam*",
"*Karim* <u>gives</u> a loaf of *bread* to *Abu*",
"*Aminah* <u>gives</u> a *lecture* by *Mariam*"

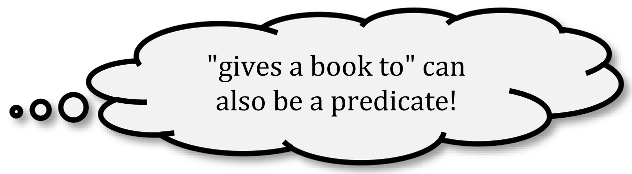


are obtained by substituting an appropriate object for variables x, y, and z in the sentence "x gives y to z".

The template "... gives... to..." is a predicate and it describes a relationship among three objects. This predicate can be represented by Give(x, y, z) or G(x, y, z), for example.

Note: The sentence

"John gives the book to Mary"



can also be represented by another predicate such as "gives a book to".

Thus, if we use B(x, y) to denote this predicate, "John gives the book to Mary" becomes B(John, Mary). In that case, the other sentences for example "Jim gives a loaf of bread to Tom", and "Jane gives a lecture to Mary", must be expressed with other predicates.

Exercise 12:

1. Let G(x, y) represents the predicate $x > y$. State whether the following predicates are true or false.
   a. G(6,13) means 13 is greater than 6 (True/False)
   b. G(2,0) (True/False)
   c. G(7,1) means 7 is greater than 1 (True/False)
   d. "4 is less than 5" can be represented by G(5,4) (True/False)

2. Let E(x, y) represents "x sent an email to y". State whether the following predicates are true or false.
   a. ~E(A,B) means A did not sent email to B (True/False)
   b. E(A,B) = E(B,A) (True/False)
   c. "B sent an email to A" is represented by E(B,A) (True/False)
   d. E(x, y) can also be represented by a 3 variable predicate (True/False)

3. Let $Q(x, y)$ denote the statement "x is greater than y." What are the truth values of the following?
   a. $Q(3,1)$
   b. $Q(5,5)$
   c. $Q(6,-6)$
   d. $Q(2^8,256)$

3) Compound Statement in Predicate Logic

Identify the compound statement in predicate logic where the compound statements obtained via logical connectives. Statement P(x) may not be a proposition since there are more objects it can be applied to. This is the same as in propositional logic but the difference is predicate logic allows us to explicitly manipulate and substitute for the objects.

Exercise 13:

State whether the following predicates are proposition or not.

1. Student(Siti) $\wedge$ Student(Nur)

2. Country(Sienna) $\vee$ River(Sienna)

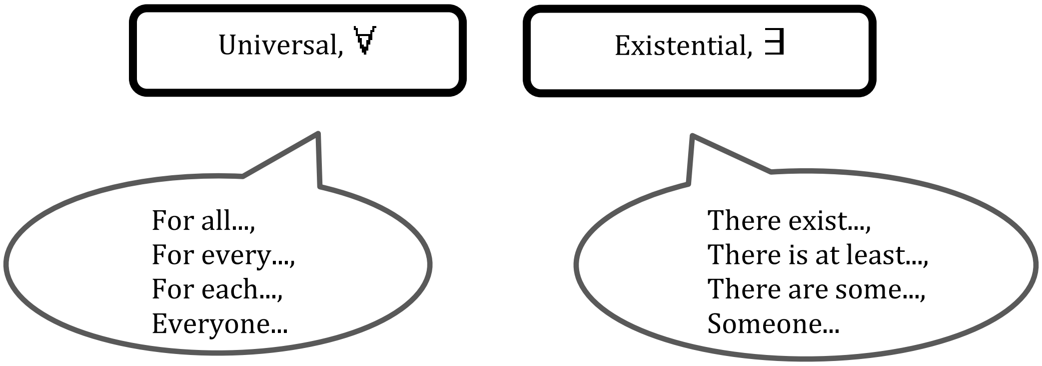
3. CS-major(x) $\rightarrow$ Student(x)

3.2.2 QUANTIFIERS

1) Type of Quantifiers

There are two types of quantifier that being used in this course:

a. Universal quantifier
b. Existential quantifier



2) Quantified Statements

Predicate logic permits quantified sentences where variables are substituted for statements about the group of objects.

Again, the statement $x > 1$ is not a proposition. It can be true or false depending on the value of x. For $x > 1$ to be a proposition either we substitute a specific number for x or change it to something like:

a. "There is a number x for which $x > 1$ holds" **($\exists x\ x > 1$)**
b. "For every number x, $x > 1$ holds" **($\forall x\ x > 1$)**

More generally, a predicate with variables (called an atomic formula) can be made a proposition by applying one of the following two operations to each of its variables:

a. Assign a value to the variable
b. Quantify the variable using a quantifier

Exercise 14:

1. Let P(x) be the statement " $x > x^2$." If the universe of discourse is the set of real numbers, what are the truth values of the following?

   a. P(0)
   b. P(2)
   c. P(-1)
   d. $\exists x$ P(x)
   e. $\forall x$ P(x)

3) Predicate Logic in English Sentences

When reading quantified formulas in English, read them from left to right. For example, let the universe be the set of airplanes and let **F(*x, y*)** denote "***x*** flies faster than ***y***". Then:

a. $\forall x \forall y$ **F(*x, y*)** read as "Every airplane is faster than every airplane".

b. $\forall x \exists y$ **F(*x, y*)** read as "Every airplane is faster than some airplane".

c. $\exists x \forall y$ **F(*x, y*)** read as "Some airplane is faster than every airplane".

d. $\exists x \exists y$ **F(*x, y*)** read as "Some airplane is faster than some airplane".

Exercise 15:

1. Let P(x) be the statement "*x* is shy," where the universe of discourse for *x* is the set of students. Express each of the following quantifications in English.

   a. $\exists x\, P(x)$

   b. $\forall x\, \neg P(x)$

   c. $\exists x\, \neg P(x)$

   d. $\neg \forall x\, \neg P(x)$

Exercise 16:

1. Let L(x, y) be the predicate "x likes y," and let the universe of discourse be the set of all people. Use quantifiers to express each of the following statements.

   a. Everyone likes everyone.

   b. Someone does not like anyone.

   c. Everyone likes George.

   d. Everyone does not like someone.

2. Let E(x) = x is even and G(x, y) = $x > y$. Let the universe be the set of natural numbers.

   i. State whether each of the following are true or false.

   a. $\exists x$ E(x)

   b. $\forall x \forall y$ G(x, y)

   ii. Write this predicate logic to English sentences.

   a. $\forall x \exists y$ G(x, y)

   b. $\exists x \forall y$ G(x, y)

EXERCISES

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
   a. Ipoh is a capital city of Selangor
   b. Shah Alam is a capital city of Selangor
   c. $3 + 3 = 9$
   d. $3 + 7 = 10$
   e. Write the answer
   f. What time is it?
   g. $4 + x = 5$
   h. The moon is made of green cheese
   i. $2^n > 100$

2. What is the negation of each of these propositions?
   a. Today is Thursday
   b. There is no pollution in Kuala Lumpur
   c. $2 + 1 = 3$
   d. The weather in Malaysia is hot and sunny

3. Let p and q be the propositions "Swimming at the Port Dickson shore is allowed" and "Sharks have been spotted near the shore" respectively. Express each of these compound propositions as an English sentence.
   a. $\sim q$
   b. $p \wedge q$
   c. $\sim p \vee q$
   d. $p \rightarrow \sim q$
   e. $\sim q \rightarrow p$
   f. $\sim p \rightarrow \sim q$
   g. $p \leftrightarrow \sim q$
   h. $\sim p \wedge (p \vee \sim q)$

4. Construct a truth table for each of these compound propositions:
   a. $p \wedge \sim p$
   b. $(p \vee \neg q) \rightarrow q$
   c. $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
   d. $(p \rightarrow q) \rightarrow (q \rightarrow p)$
   e. $p \rightarrow \neg q$
   f. $(p \rightarrow q) \vee (\sim p \rightarrow q)$
   g. $(p \leftrightarrow q) \vee (\sim p \leftrightarrow q)$
   h. $(\sim p \leftrightarrow \sim q) \leftrightarrow (p \leftrightarrow q)$
   i. $\sim p \leftrightarrow q$
   j. $(p \rightarrow q) \wedge (\sim p \rightarrow q)$
   k. $(p \vee q) \vee r$
   l. $(p \wedge q) \vee r$
   m. $p \vee \neg p$
   n. $(p \vee q) \rightarrow (p \wedge q)$
   o. $(p \vee q) \wedge \neg r$
   p. $(p \vee q) \wedge r$
   q. $(p \wedge q) \wedge r$
   r. $(p \wedge q) \vee \sim r$
   s. $p \rightarrow (\sim q \vee r)$
   t. $\sim p \rightarrow (q \rightarrow r)$
   u. $(p \rightarrow q) \vee (\sim p \rightarrow r)$
   v. $(p \rightarrow q) \wedge (\sim p \rightarrow r)$
   w. $(p \leftrightarrow q) \vee (\sim q \leftrightarrow r)$
   x. $(\sim p \leftrightarrow \sim q) \vee (q \leftrightarrow r)$

5. Determine whether these conditional statements are tautology, contingency or contradiction:
   a. $(p \wedge q) \to p$
   b. $\sim p \to (p \to q)$
   c. $\neg(p \to q) \to p$
   d. $p \to (p \vee q)$
   e. $(p \wedge q) \to (p \to q)$
   f. $\sim (p \to q) \to \sim q$
   g. $[\sim p \wedge (p \vee q)] \to q$
   h. $[(p \to q) \wedge (q \to r)] \to (p \to r)$
   i. $[p \wedge (p \to q)] \to q$
   j. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
   k. $(\sim p \wedge (p \to q)) \to \sim q$
   l. $(\sim q \wedge (p \to q)) \to \sim p$

6. Show that the following conditional statements are logically equivalent or not:
   a. $p \leftrightarrow q \,\&\, (p \wedge q) \vee (\sim p \wedge \sim q)$
   b. $\sim (p \leftrightarrow q) \,\&\, p \leftrightarrow \sim q$
   c. $(p \to q) \wedge (p \to r) \,\&\, p \to (q \wedge r)$
   d. $(p \to r) \wedge (q \to r) \,\&\, (p \vee q) \to r$
   e. $(p \to q) \vee (p \to r) \,\&\, p \to (q \vee r)$
   f. $(p \to r) \vee (q \to r) \,\&\, (p \wedge q) \to r$
   g. $\sim p \to (q \to r) \,\&\, q \to (p \vee r)$
   h. $p \leftrightarrow q \,\&\, (p \to q) \wedge (q \to p)$
   i. $p \leftrightarrow q \,\&\, \sim p \leftrightarrow \sim q$
   j. $(p \to q) \wedge (q \to r) \,\&\, \sim p \leftrightarrow \sim q$
   k. $(p \to q) \to r \,\&\, p \to (q \to r)$
   l. $(p \wedge q) \to r \,\&\, (p \to r) \wedge (q \to r)$

7. Let T(x,y) mean x is taller than y and let the universe be set of people.
   a. $\forall x$ T(x,y) (True/False)

   b. $\forall x \forall$y T(x,y) (True/False)

   c. $\forall xy$ T(x,y) (True/False)

   d. Write this predicate logic to English sentences.
      i. $\forall x \exists y$ T(x,y)
      ii. $\exists x \forall y$ T(x,y)