POPULATION GENETICS: Hardy-Weinberg Law

The Hardy-Weinberg states that under a given set of conditions, the allele and genotype frequencies in a population do not change and remains constant from one generation to next. It is called the **Hardy-Weinberg Equilibrium**. What are the conditions? What does the principle mean? And how do you solve those Hardy-Weinberg problems? Everything shall be revealed later.

The Hardy-Weinberg Equilibrium is applicable if certain conditions are met in a population. The five conditions that must be met for genetic equilibrium to occur are:

1. **Large population size** : the population is so large that allele frequencies do not change due to random chance.
2. **Random mating** : each individuals in a population has an equal chance of mating with any individual of the opposite sex.
3. **No mutations** : no biochemical changes in DNA that produce new alleles.
4. **No migration** : individuals do not travel between different populations.
5. **No natural selection** : all organisms survive and have same reproduction potential.

So do you understand? Not yet!!! OK, OK.. When a population is in Hardy-Weinberg equilibrium for a gene, it is not evolving, and allele frequencies will stay the same across generations. Therefore, it allows us to predict genotype frequencies in living populations.

To understand this law even better, we will see the expression of these alleles and come up with equations. Let's return to our hypothetical rabbit example in which a gene is polymorphic and exists as two different alleles: **B** and **b**.

The frequency of dominant allele will be represented by **p** and the frequency recessive allele will be represented by **q**. So, as I had mentioned earlier, the sum of all frequencies must equal 1. Therefore,

$$p + q = 1$$

From the example, if p = 0.3, then q must be 0.7. In other words, if the allele frequency of **B** equals 30%, the remaining 70% of alleles must be **b**, because together they equal 100%.

How to predict genotype frequencies? I know you must be asking for that. If a population is in Hardy-Weinberg equilibrium, allele frequencies will be related to genotype frequencies by a specific mathematical relationship, the **Hardy-Weinberg equation**.

We know genotype is two alleles. It means you have to multiply allele number one ($p + q = 1$) and allele number two ($p + q = 1$). So both of those are equals to 1. Therefore, by using the product rule and multiply those out, you will get:

$$(p + q)(p + q) = 1$$

$$pp + qp + pq + qq = 1$$

$$p^2 + 2pq + q^2 = 1$$ Hardy-Weinberg Equation

And this is going to be the key for Hardy-Weinberg Equilibrium in which:

p^2 = frequency of homozygous dominant genotype
$2pq$ = frequency of heterozygous genotype
q^2 = frequency for homozygous recessive genotype

If the Hardy-Weinberg equation is related to our hypothetical rabbit population in which gene exists in alleles designated **B** and **b**, then

p^2 = genotype frequency of **BB**
$2pq$ = genotype frequency of **Bb**
q^2 = genotype frequency of **bb**

So, we can predict the genotype frequencies (if the population is in Hardy-Weinberg equilibrium) by plugging in allele frequencies as shown below:

Given $p = 0.3$ and $q = 0.7$, then

$BB = p^2 = (0.3)^2 = 0.09 \approx 0.1$

$Bb = 2pq = 2\,(0.3)(0.7) = 0.42 \approx 0.4$

$bb = q^2 = (0.7)^2 = 0.49 \approx 0.5$

Let's imagine that these are, in fact, the genotype frequencies we see in our rabbits population (10% **BB**, 40% **Bb**, 50% **bb**). Excellent—our rabbits appear to be in Hardy-Weinberg equilibrium! Now, let's imagine that the rabbits reproduce to make a next generation. What will the allele and genotype frequencies will be in that generation?

Hardy-Weinberg Principle

Parent Generation

Phenotypes			
Genotypes	**BB**	**Bb**	**bb**
Number of Rabbits (Total = 20)	**2**	**8**	**10**
Genotype Frequencies	$\frac{2}{20} = 0.1\ BB$	$\frac{8}{20} = 0.4\ Bb$	$\frac{10}{20} = 0.5\ bb$
Number of alleles in gene pool (Total = 40)	**X 2** **4 B**	**8 B** **8 b**	**X 2** **20 b**
Allele Frequencies	$\frac{12}{40} = 0.3\ B$		$\frac{28}{40} = 0.7\ b$

Hardy Weinberg Analysis

	B (0.3)	**b** (0.7)
B (0.3)	**BB** $(0.3)^2 = 0.09$ $= 0.1$	**Bb** (0.3)(0.7) = 0.21
b (0.7)	**Bb** (0.3)(0.7) = 0.21	**bb** $(0.7)^2 = 0.49$ $= 0.5$

p^2	**+**	**2pq**	**+**	**q^2**	**=**	**1**
$(0.3)^2$	+	2(0.3)(0.7)	+	$(0.7)^2$	=	1
$0.09 \approx 0.1$	+	$0.42 \approx 0.4$	+	$0.49 \approx 0.5$	=	1
Predicted frequency of BB offspring		**Predicted frequency of Bb offspring**		**Predicted frequency of bb offspring**		

As shown above, we'd predict an offspring generation with the exact same genotype frequencies as the parent generation: 10% **BB**, 40% **Bb**, 50% **bb**. If genotype frequencies have not changed, we also must have the same allele frequencies as in the parent generation: 0.3 for **B** and 0.7 for **b**. We're back where we started. No evolution of population.

Solving Hardy-Weinberg Problems

Let's look at some more examples..

In a population of 1000 individuals which is at genetic equilibrium, 840 individuals have dominant brown eyes while the rest have recessive blue eyes. Find the frequency of brown allele, the blue allele and the frequencies of the three possible genotypes in this population.

The key to Hardy-Weinberg problems lies in the homozygous recessive individuals. Before anything, **always find the homozygous recessive individuals first**. The reason why because those individuals can be counted. You can't tell the difference between the homozygous dominant and the heterozygous individuals because both represent as dominant gene. But you can see the homozygous recessive ones.

Out of 1000 individuals, 840 are dominant gene. Therefore, the number of homozygous recessive individuals = 1000 – 840 = 160. Remember..this is q^2 NOT **q**.

$$q^2 = \frac{160}{1000} = 0.16$$

$$q = \sqrt{0.16} = 0.40$$

So, once you have **q**, you can easily get **p** because p + q = 1. Thus,

p = 1 – 0.40 = 0.60

Hence, the frequency for brown allele is 0.60 and the frequency of blue allele is 0.40.

Once you have **p** and **q**, you can calculate all the other genotype frequencies by using $p^2 + 2pq + q^2 = 1$.

Frequency of homozygous dominant genotype = $p^2 = (0.60)^2 = 0.36$
Frequency for homozygous recessive genotype = $q^2 = (0.40)^2 = 0.16$
Frequency of heterozygous genotype = 2pq = 2 (0.60)(0.40) = 0.48

And that's it!

Let's try another one. Now it's your turn to do.

The ability to taste PTC is controlled by a single gene T. Thirty percent of the population is unable to taste the chemical PTC. These non-tasters are recessive for the tasting gene. Assuming the population is in Hardy-Weinberg equilibrium, calculate:

i. the frequency of dominant and recessive allele.
ii. the frequency for heterozygous for the trait.
iii. the frequency for non-taster for next generation.

Answer:

i. $q^2 = \frac{30}{100} = 0.30$

$q = \sqrt{0.30} = 0.55$

p + q = 1
p = 1 – 0.55
p = 0.45

Therefore, the frequency for T allele is 0.45 and the frequency for t allele is 0.55.

ii. Frequency of heterozygous genotype = 2pq = 2 (0.45)(0.55) = 0.495 ≈ 0.50 (round to 2 decimal places)

iii. There are no changes in the frequencies in the next generation as the population is in equilibrium. Therefore, the frequency for homozygous recessive genotype = $q^2 = (0.55)^2 = 0.3025 \approx 0.30$**.**