PRINCIPLES OF STEADY HEAT TRANSFER IN CONVECTION

CHAPTER 3

FUNDAMENTAL OF CONVECTION

OBJECTIVES

When you finish studying this chapter, you should be able to;

- Understand the physical mechanism of convection, and its classification
- Visualize the development of velocity and thermal boundary layers during flow over surfaces
- Gain a working knowledge of the dimensionless Reynolds, Prandtl and Nusselt numbers
- Distinguish between laminar and turbulent flows, gain an understanding of the mechanisms of momentum and heat transfer in turbulent flow.

PHYSICAL MECHANISM OF CONVECTION

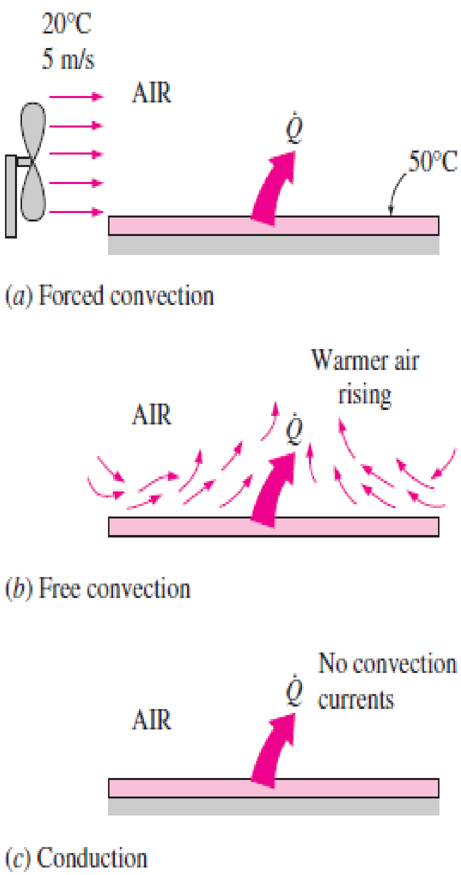


FIGURE 6–1
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

- Conduction and convection are similar in that both mechanisms require the presence of a material medium.
- But they are different in that convection requires the presence of fluid motion.
- Heat transfer through a liquid or gas can be by conduction or convection, depending on the presence of any bulk fluid motion.
- Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it.

- consider steady heat transfer through a fluid contained between two parallel plates maintained at different temperatures, as shown in Figure 6-2
- Assuming no fluid motion, the energy of the hotter fluid molecules near the hot plate will be transferred to the adjacent cooler fluid molecules. This is what happens during conduction through a fluid.
- Now let us use a syringe to draw some fluid near the hot plate and inject it near the cold plate repeatedly. You can imagine that this will speed up the heat transfer process considerably, since some energy is carried to the other side as a result of fluid motion.

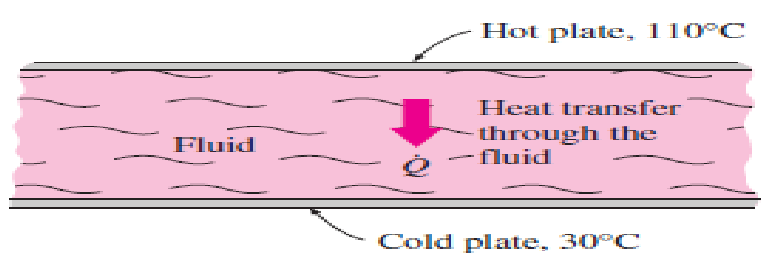


FIGURE 6–2
Heat transfer through a fluid sandwiched between two parallel plates.

- Consider the cooling of a hot iron block with a fan blowing air over its top surface, as shown in Figure 6–3
- Heat will be transferred from the hot block to the surrounding cooler air, and the block will eventually cool.
- The block will cool faster if the fan is switched to a higher speed. Replacing air by water will enhance the convection heat transfer even more.

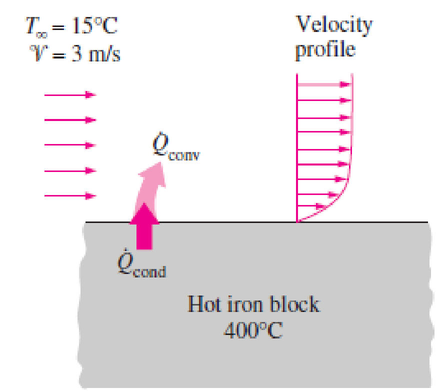


FIGURE 6–3
The cooling of a hot block by forced convection.

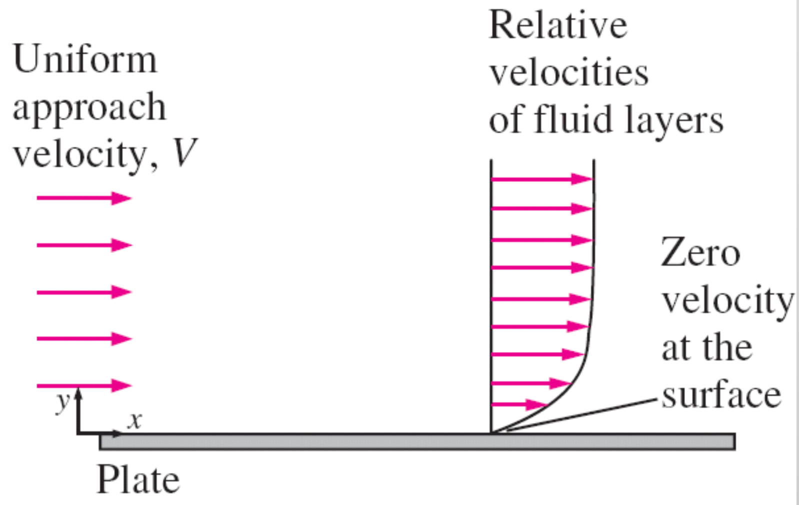
- Experience shows that convection heat transfer strongly depends on the fluid properties:
  - dynamic viscosity μ,
  - thermal conductivity k,
  - density ρ, and
  - specific heat c_p, as well as the
  - fluid velocity V.
- It also depends on the *geometry* and the *roughness* of the solid surface.
- The rate of convection heat transfer is observed to be proportional to the temperature difference and is expressed by **Newton's law of cooling** as
-

$$\dot{q}_{conv} = h\left(T_s - T_\infty\right) \quad (\mathrm{W/m^2}) \qquad (1)$$

or

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \quad (W) \qquad (2)$$

- All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface (**no-slip**).
- The **no-slip** condition is responsible for the development of the velocity profile.
- The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer**.



- An implication of the no-slip condition is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction*, and can be expressed as

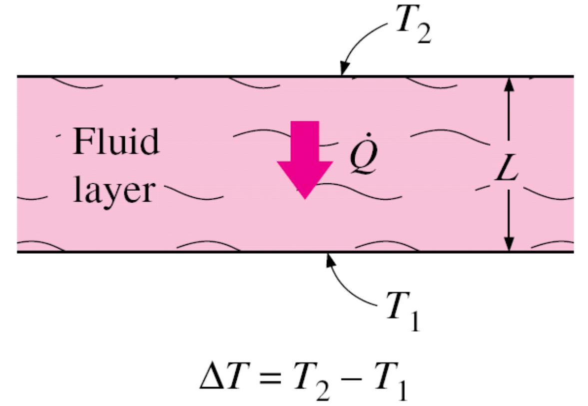
$$\dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid} \left. \frac{\partial T}{\partial y} \right|_{y=0} \qquad (\mathrm{W/m^2}) \tag{3}$$

- Equating Eqs. 1 and 3 for the heat flux to obtain
- $$h = \frac{-k_{fluid} \left(\partial T / \partial y \right)_{y=0}}{T_s - T_\infty} \qquad (\mathrm{W/m^2 \cdot {}^\circ C}) \tag{4}$$
- The convection heat transfer coefficient, in general, varies along the flow direction.

Nusselt Number

- It is common practice to nondimensionalize the heat transfer coefficient *h* with the Nusselt number $$Nu = \frac{hL_c}{k} \qquad (5)$$
- Heat transfer through the fluid layer will be by *convection* when the fluid involves some motion and by *conduction* when the fluid layer is motionless.
- Heat flux (the rate of heat transfer per unit time per unit surface area) in either case will be $$\dot{q}_{cond} = k\frac{\Delta T}{L} \qquad (7)$$
- $$\dot{q}_{conv} = h\Delta T \qquad (6)$$
- Taking their ratio gives

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\Delta T}{k\Delta T / L} = \frac{hL}{k} = Nu$$



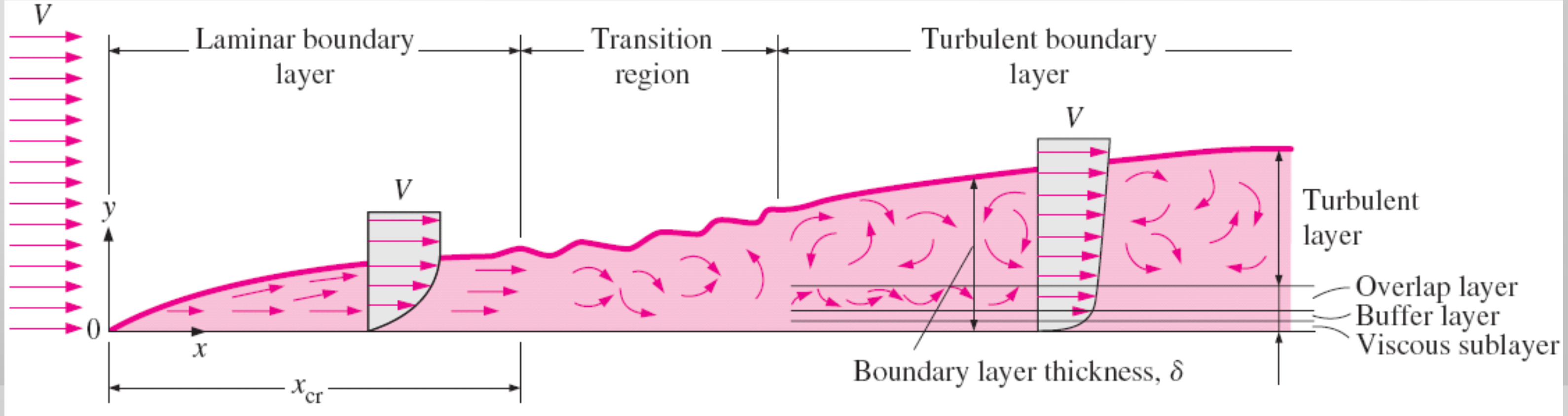
- The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.
- Nu=1 →pure conduction. **Nusselt number** (**Nu**) is the ratio of convective to conductive heat transfer across (normal to) the boundary

Classification of Fluid Flows

- Viscous versus inviscid regions of flow An **inviscid flow** is the flow of an ideal fluid that is assumed to have no viscosity. In fluid dynamics there are problems that are easily solved by using the simplifying assumption of an inviscid flow.
- Internal versus external flow
- Compressible versus incompressible flow
- Laminar versus turbulent flow
- Natural (or unforced) versus forced flow
- Steady versus unsteady flow
- One-, two-, and three-dimensional flows

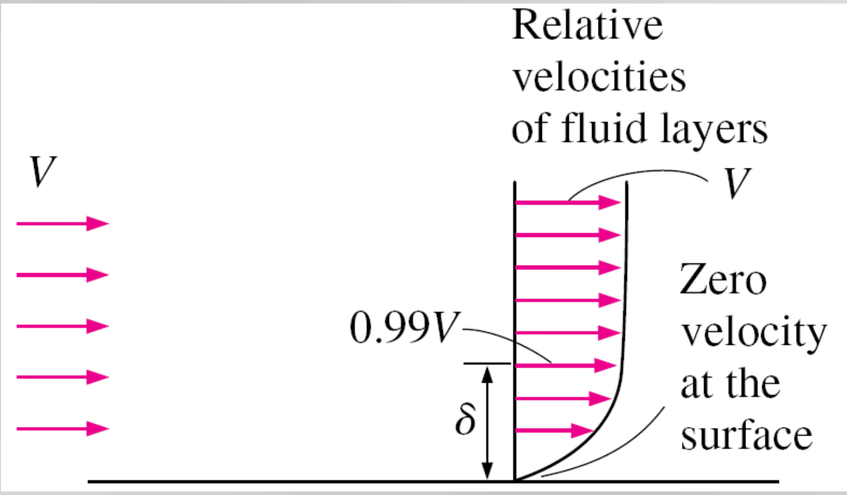
Velocity Boundary Layer

- Consider the parallel flow of a fluid over a *flat plate.*
- *x*-coordinate: along the plate surface
- *y*-coordinate: from the surface in the normal direction.
- The fluid approaches the plate in the *x*-direction with a uniform velocity *V*.
- Because of the no-slip condition $V(y=0)=0$.
- The presence of the plate is felt up to δ.
- Beyond δ the free-stream velocity remains essentially unchanged.
- The fluid velocity, *u*, varies from 0 at $y=0$ to nearly *V* at $y=\delta$.



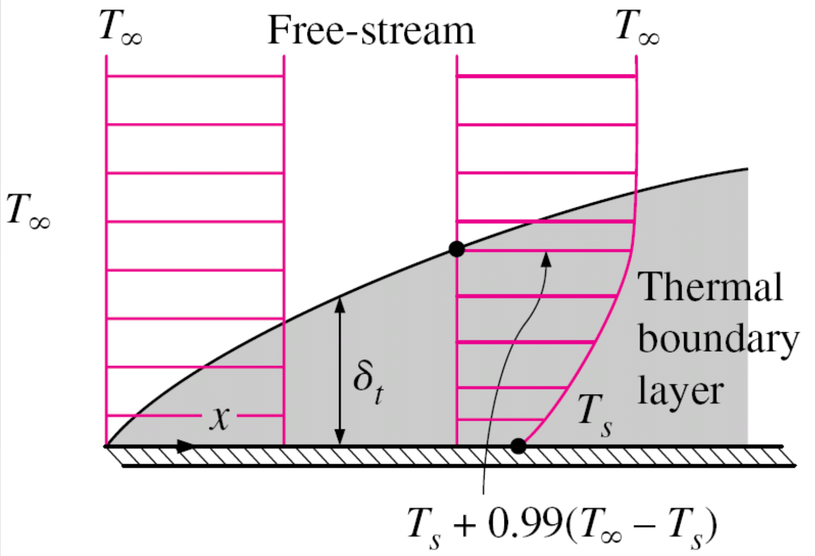
Velocity Boundary Layer

- The region of the flow above the plate bounded by δ is called the **velocity boundary layer**.
- δ is typically defined as the distance y from the surface at which $u=0.99V$.
- The hypothetical line of $u=0.99V$ divides the flow over a plate into two regions:
  - the **boundary layer region**, and
  - the **irrotational flow region**.



Thermal Boundary Layer

- Like the velocity a *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature.



- Consider the flow of a fluid at a uniform temperature of T_∞ over an isothermal flat plate at temperature T_s.
- The fluid particles in the layer adjacent assume the surface temperature T_s.
- A temperature profile develops that ranges from T_s at the surface to T_∞ sufficiently far from the surface.
- The **thermal boundary layer** – the flow region over the surface in which the temperature variation in the direction normal to the surface is significant.

Thermal Boundary Layer

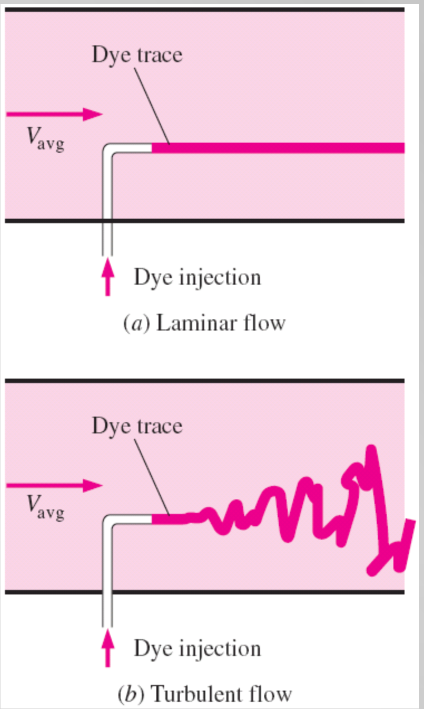
- The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference* $T(y=\delta_t)-T_s= 0.99(T_\infty-T_s)$.
- The thickness of the thermal boundary layer increases in the flow direction.
- The convection heat transfer rate anywhere along the surface is directly related to the temperature gradient at that location.

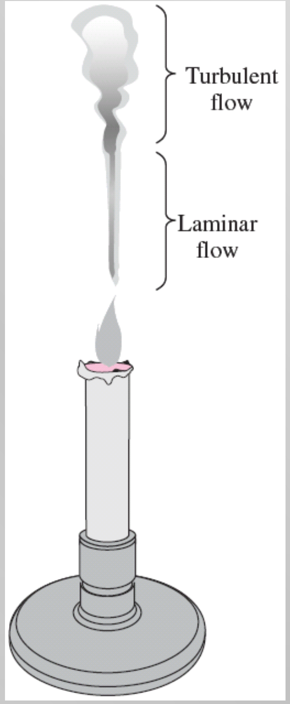
Prandtl Number

- The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number**, defined as
- $$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k} \qquad (8)$$
- Heat diffuses very quickly in liquid metals (Pr«1) and very slowly in oils (Pr»1) relative to momentum.
- Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.
- The **Prandtl number** is a dimensionless number; the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. It is named after the German physicist Ludwig Prandtl.

Laminar and Turbulent Flows

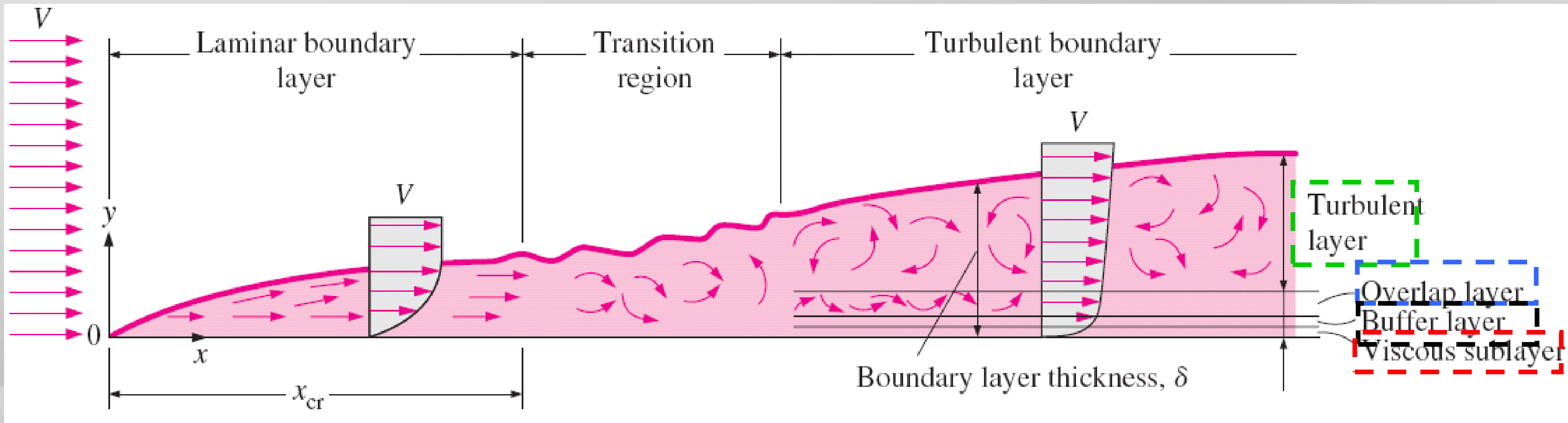
- Laminar flow – the flow is characterized by *smooth streamlines* and *highly-ordered motion.*
- Turbulent flow – the flow is characterized by *velocity fluctuations* and *highly-disordered motion.*
- The **transition** from laminar to turbulent flow does not occur suddenly.





Laminar and Turbulent Flows

- The velocity profile in turbulent flow is much fuller than that in laminar flow, with a sharp drop near the surface.
- The turbulent boundary layer can be considered to consist of four regions:
  - Viscous sublayer
  - Buffer layer
  - Overlap layer
  - Turbulent layer
- The *intense mixing* in turbulent flow enhances heat and momentum transfer, which increases the friction force on the surface and the convection heat transfer rate.



Reynolds Number

- The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, flow velocity, surface temperature*, and *type of fluid.*
- The flow regime depends mainly on the ratio of the *inertia forces* to *viscous forces* in the fluid.
- This ratio is called the **Reynolds number**, which is expressed for external flow as

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{VL_c}{\nu} = \frac{\rho VL_c}{\mu}$$

- At *large* Reynolds numbers (turbulent flow) the inertia forces are large relative to the viscous forces.
- At *small* or *moderate* Reynolds numbers (laminar flow), the viscous forces are large enough to suppress these fluctuations and to keep the fluid "inline."
- **Critical Reynolds number** – the Reynolds number at which the flow becomes turbulent.

HEAT TRANSFER IN CONVECTION

EXTERNAL FORCED CONVECTION

OBJECTIVES:

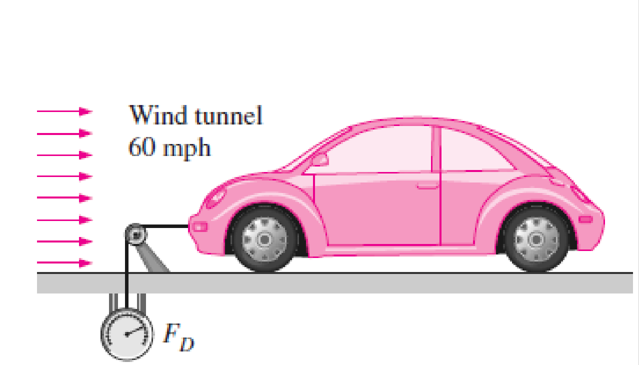
When you finish studying this topic, you should be able to;

- Distinguish between internal and external flow
- Develop an intuitive understanding of friction drag and pressure drag, and evaluate the average drag and convection coefficients in external flow
- Evaluate the drag and heat transfer associated with flow over a flat plate for both laminar and turbulent flow
- Calculate the drag force exerted on cylinders during cross flow, and the average heat transfer coefficient

DRAG AND HEAT TRANSFER IN EXTERNAL FLOW

- Fluid flow over solid bodies frequently occurs in practice, and it is responsible for numerous physical phenomena such as the *drag force* acting on the automobiles, power lines, trees, and underwater pipelines

FRICTION AND PRESSURE DRAG



- Experience when you extend your arm out of the window of a moving car.
- The force a flowing fluid exerts on a body in the flow direction is called **drag**

- The drag force F_D depends on the density of the fluid, the upstream velocity, and the size, shape, and orientation of the body, among other things.
- The drag characteristics of a body is represented by the dimensionless **drag coefficient** C_D defined as

Drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathcal{V}^2 A} \qquad (7\text{-}1)$$

- where A is the *frontal area* (the area projected on a plane normal to the direction of flow) for blunt bodies—bodies that tends to block the flow
- The frontal area of a cylinder of diameter D and length L, for example, is $A = LD$.
- For parallel flow over flat plates or thin airfoils, A is the surface area.

- Flat Plate:

$$C_{D,\,\text{pressure}} = 0$$
$$C_D = C_{D,\,\text{friction}} = C_f$$

$$F_{D,\,\text{pressure}} = 0$$
$$F_D = F_{D,\,\text{friction}} = F_f = C_f A \frac{\rho \mathcal{V}^2}{2}$$

FIGURE 7–3
For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

- The fluid temperature in the thermal boundary layer varies from T_s at the surface to about T_∞ at the outer edge of the boundary.
- The fluid properties also vary with temperature, and thus with position across the boundary layer.
- So the fluid properties are usually evaluated at the so called film temperature, defined as

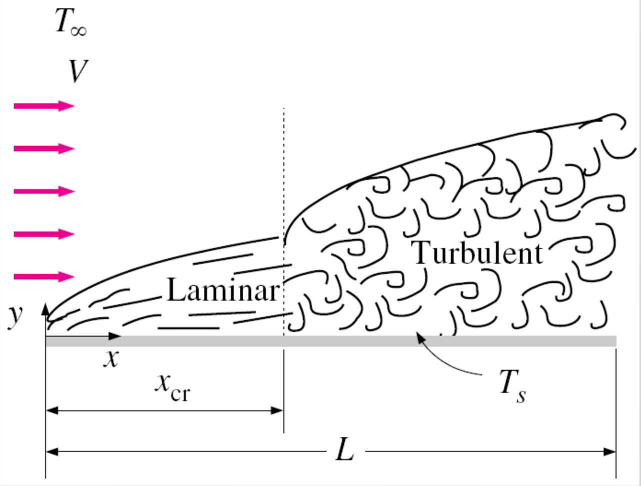
$$T_f = \frac{T_s + T_\infty}{2}$$

- Which is the arithmatic average of the surface and the free stream temperatures.

PARALLEL FLOW OVER FLAT PLATES

- Consider the **parallel flow** of a fluid over a flat plate of **length** *L* in the flow direction.
- The Reynolds number at a **distance** *x* from the leading edge of a flat plate is expressed as

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$$



- In engineering analysis, a generally **accepted value** for the **critical** Reynolds number is

$$\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} = 5\times 10^5$$

- The **actual value** of the engineering critical Reynolds number may vary somewhat from 10^5 to 3×10^6.

Friction Coefficient

- The average friction coefficient over the entire plate is determined by:

Laminar: $$C_f = \frac{1.328}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

Average Nusselt Number

- The *average* Nusselt number
  - *Laminar:* $Nu = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re} < 5\times10^5$
  - *Turbulent:* $Nu = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$, $\quad 5\times10^5 \le \mathrm{Re}_x \le 10^7$

- In some cases, a flat plat is sufficiently long for the flow to become turbulent. But not long enough to disregard the laminar flow region.

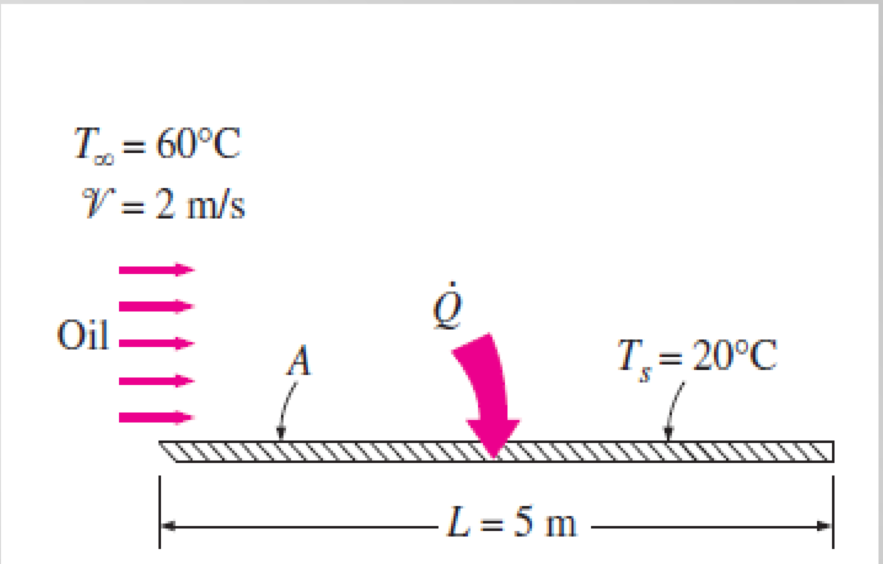
$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\ \mathrm{laminar}}\,dx + \int_{x_{cr}}^{L} h_{x,\ \mathrm{turbulent}}\,dx\right)$$

$$\mathrm{Re}_{cr} = 5\times10^5$$

$$Nu = \left(0.037\,\mathrm{Re}_L^{0.8} - 871\right)\mathrm{Pr}^{1/3}$$

Example 1 Flow of Hot Oil over a Flat Plate

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s . Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



SOLUTION Engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$.

Properties The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40°C$ are (Table A–14).

$$\rho = 876 \text{ kg/m}^3 \qquad \Pr = 2870$$
$$k = 0.144 \text{ W/m} \cdot °\text{C} \qquad \nu = 242 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis Noting that $L = 5$ m, the Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2/\text{s}} = 4.13 \times 10^4$$

which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is

$$C_f = 1.328 \, \text{Re}_L^{-0.5} = 1.328 \times (4.13 \times 10^3)^{-0.5} = 0.0207$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the plate per unit width becomes

$$F_D = C_f A_s \frac{\rho \mathcal{V}^2}{2} = 0.0207 \times (5 \times 1 \text{ m}^2) \frac{(876 \text{ kg/m}^3)(2 \text{ m/s})^2}{2} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$
$$= \mathbf{181 \text{ N}}$$

Similarly, the Nusselt number is determined using the laminar flow relations for a flat plate,

$$\text{Nu} = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

Then,

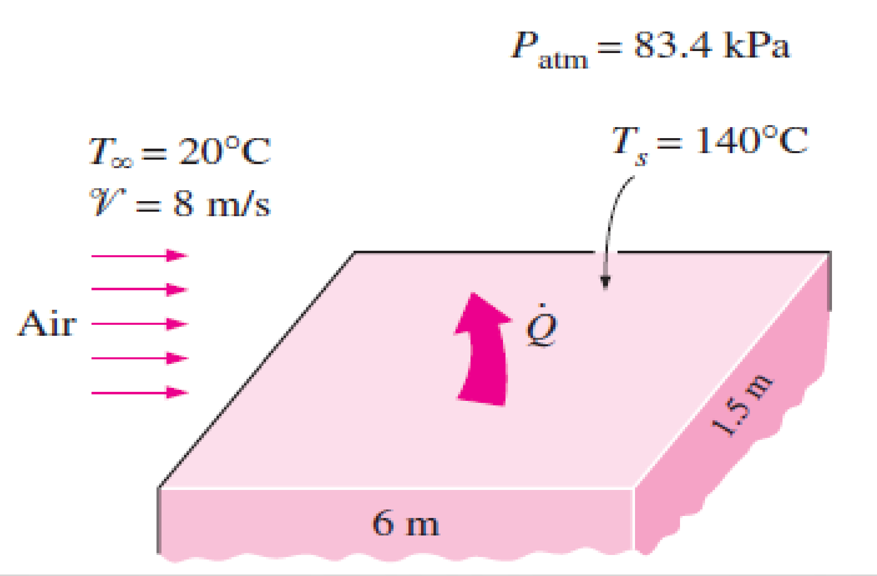
$$h = \frac{k}{L}\text{Nu} = \frac{0.144\ \text{W/m}\cdot{}^\circ\text{C}}{5\ \text{m}}(1918) = 55.2\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2\ \text{W/m}^2\cdot{}^\circ\text{C})(5 \times 1\ \text{m}^2)(60 - 20)^\circ\text{C} = 11{,}040\ \text{W}$$

Example 2: Cooling of a Hot Block by Forced Air at High Elevation

- The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m 6 m flat plate whose temperature is 140°C (Fig. 7-13). Determine the rate of heat transfer from the plate if the air flows parallel to the (*a*) 6-m-long side



SOLUTION

Assumptions 1 Steady operating conditions exist. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5 \times 10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas.

Properties The properties k, μ, C_p, and Pr of ideal gases are independent of pressure, while the properties ν and α are inversely proportional to density and thus pressure. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (140 + 20)/2 = 80°C$ and 1 atm pressure are (Table A–15)

$$k = 0.02953 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu_{@\,1\text{ atm}} = 2.097 \times 10^{-5} \text{ m}^2/\text{s}$$

The atmospheric pressure in Denver is $P = (83.4 \text{ kPa})/(101.325 \text{ kPa/atm}) = 0.823$ atm. Then the kinematic viscosity of air in Denver becomes

$$\nu = \nu_{@\,1\text{ atm}}/P = (2.097 \times 10^{-5} \text{ m}^2/\text{s})/0.823 = 2.548 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis (*a*) When air flow is parallel to the long side, we have $L = 6$ m, and the Reynolds number at the end of the plate becomes

$$\mathrm{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8 \text{ m/s})(6 \text{ m})}{2.548 \times 10^{-5} \text{ m}^2/\text{s}} = 1.884 \times 10^6$$

which is greater than the critical Reynolds number. Thus, we have combined laminar and turbulent flow, and the average Nusselt number for the entire plate is determined to be

$$\begin{aligned} \text{Nu} = \frac{hL}{k} &= (0.037\ \text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \\ &= [0.037(1.884 \times 10^6)^{0.8} - 871]0.7154^{1/3} \\ &= 2687 \end{aligned}$$

Then

$$h = \frac{k}{L}\text{Nu} = \frac{0.02953\ \text{W/m} \cdot {}^\circ\text{C}}{6\ \text{m}}(2687) = 13.2\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

$$A_s = wL = (1.5\ \text{m})(6\ \text{m}) = 9\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2\ \text{W/m}^2 \cdot {}^\circ\text{C})(9\ \text{m}^2)(140 - 20){}^\circ\text{C} = \mathbf{1.43 \times 10^4\ W}$$

FLOW ACROSS CYLINDERS AND SPHERES

Average Heat Transfer Coefficient

- For flow over a ***cylinder*** (Churchill and Bernstein):

$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(0.4/\mathrm{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

Re·Pr>0.2

- The **fluid properties** are evaluated at the *film temperature* [$T_f=0.5(T_\infty+T_s)$].
- Flow over a ***sphere*** (Whitaker):

$$Nu_{sph} = \frac{hD}{k} = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

- The two correlations are **accurate** within **±30%.**

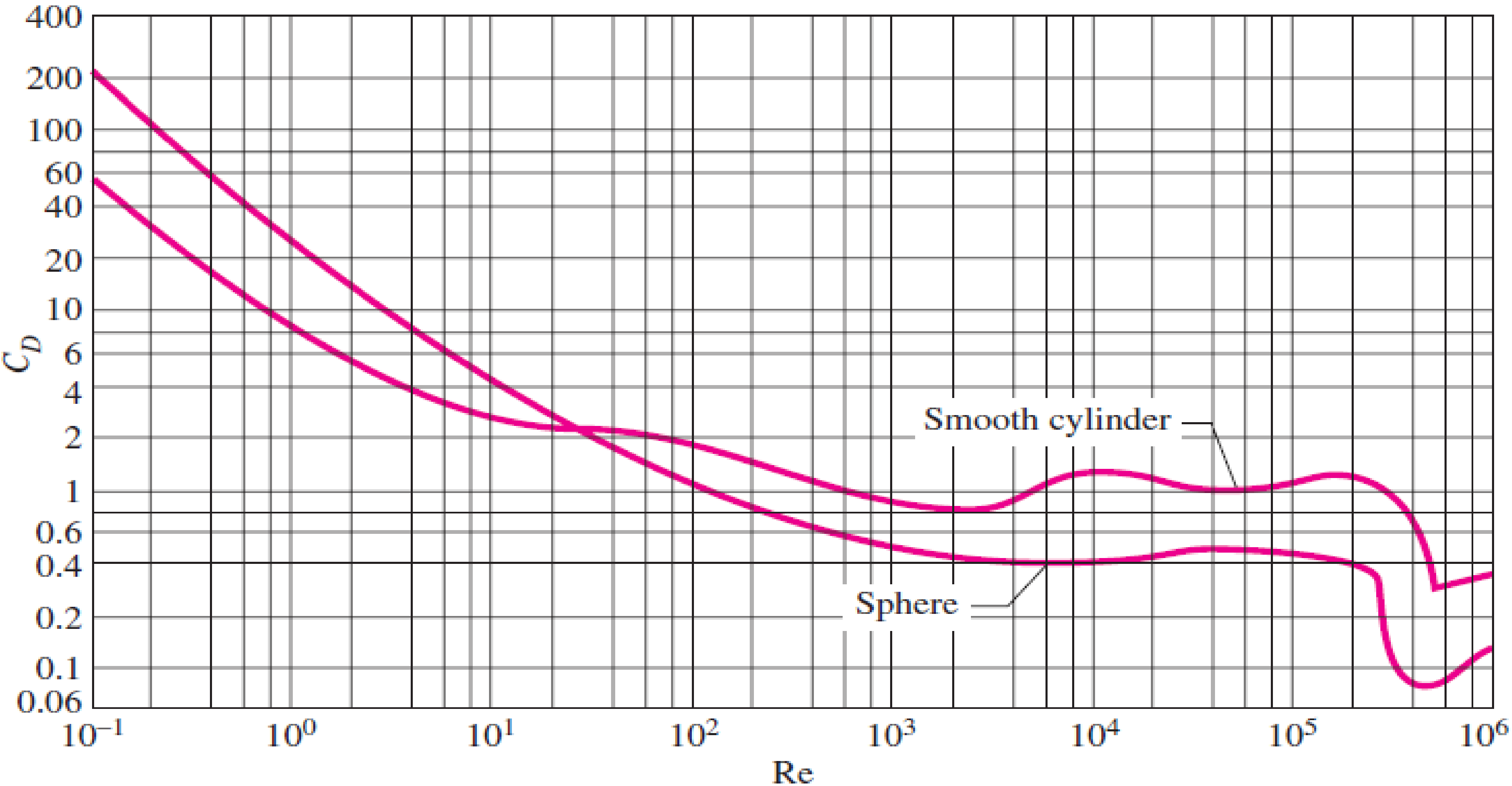
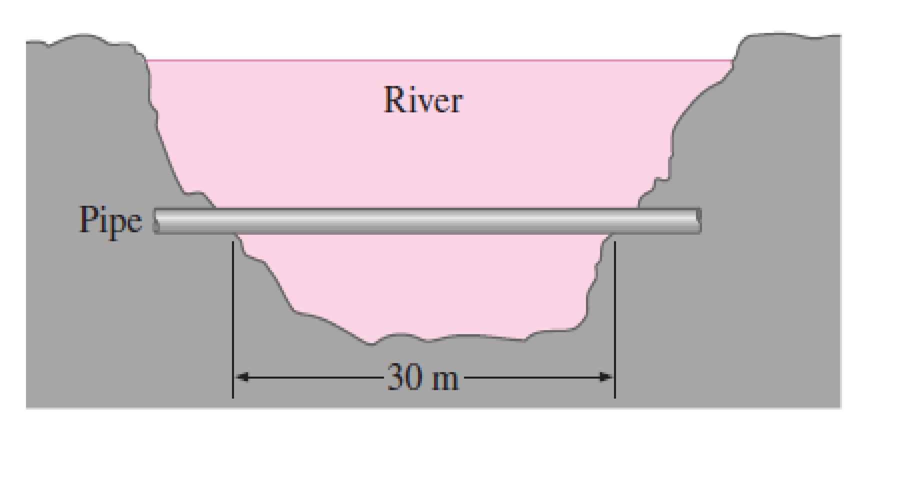


FIGURE 7–17

Average drag coefficient for cross flow over a smooth circular cylinder and a smooth sphere (from Schlichting, Ref. 10).

Example 3: **Drag Force Acting on a Pipe in a River**

- A 2.2-cm-outer-diameter pipe is to cross a river at a 30-m-wide section while being completely immersed in water . The average flow velocity of water is 4 m/s and the water temperature is 15°C.
- Determine the drag force exerted on the pipe by the river.



SOLUTION A pipe is crossing a river. The drag force that acts on the pipe is to be determined.

Assumptions **1** The outer surface of the pipe is smooth so that Figure 7–17 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The direction of water flow is normal to the pipe. **4** Turbulence in river flow is not considered.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1\ \text{kg/m}^3$ and $\mu = 1.138 \times 10^{-3}\ \text{kg/m}\cdot\text{s}$ (Table A-9).

Analysis Noting that $D = 0.022$ m, the Reynolds number for flow over the pipe is

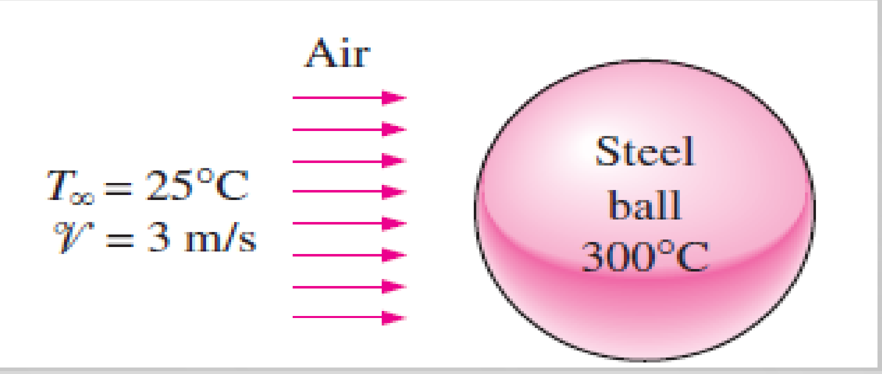
$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{\rho\mathcal{V}D}{\mu} = \frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})(0.022\ \text{m})}{1.138 \times 10^{-3}\ \text{kg/m}\cdot\text{s}} = 7.73 \times 10^4$$

The drag coefficient corresponding to this value is, from Figure 7-17, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the pipe becomes

$$F_D = C_D A \frac{\rho\mathcal{V}^2}{2} = 1.0(30 \times 0.022\ \text{m}^2)\frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= \mathbf{5275\ N}$$

Example: **Cooling of a Steel Ball by Forced Air**

- A 25-cm-diameter stainless steel ball (ρ = 8055 kg/m3, *Cp* =480 J/kg ·°C) is removed from the oven at a uniform temperature of 300°C The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C.
- Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.



Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas. **4** The outer surface temperature of the ball is uniform at all times. **5** The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

Properties The dynamic viscosity of air at the average surface temperature is $\mu_s = \mu_{@\,250°C} = 2.76 \times 10^{-5}$ kg/m · s. The properties of air at the free-stream temperature of 25°C and 1 atm are (Table A-15)

$$k = 0.02551 \text{ W/m} \cdot °\text{C} \qquad \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu = 1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr} = 0.7296$$

Analysis The Reynolds number is determined from

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(3 \text{ m/s})(0.25 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 4.802 \times 10^4$$

The Nusselt number is

$$\begin{aligned}\text{Nu} = \frac{hD}{k} &= 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\,\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \\ &= 2 + [0.4(4.802 \times 10^4)^{1/2} + 0.06(4.802 \times 10^4)^{2/3}](0.7296)^{0.4} \\ &\quad \times \left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1/4} \\ &= 135\end{aligned}$$

Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551 \text{ W/m} \cdot °\text{C}}{0.25 \text{ m}}(135) = \mathbf{13.8 \text{ W/m}^2 \cdot °\text{C}}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25\ \text{m})^2 = 0.1963\ \text{m}^2$$

$$\dot{Q}_{\text{ave}} = hA_s(T_{s,\text{ ave}} - T_\infty) = (13.8\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.1963\ \text{m}^2)(250 - 25)^\circ\text{C} = 610\ \text{W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

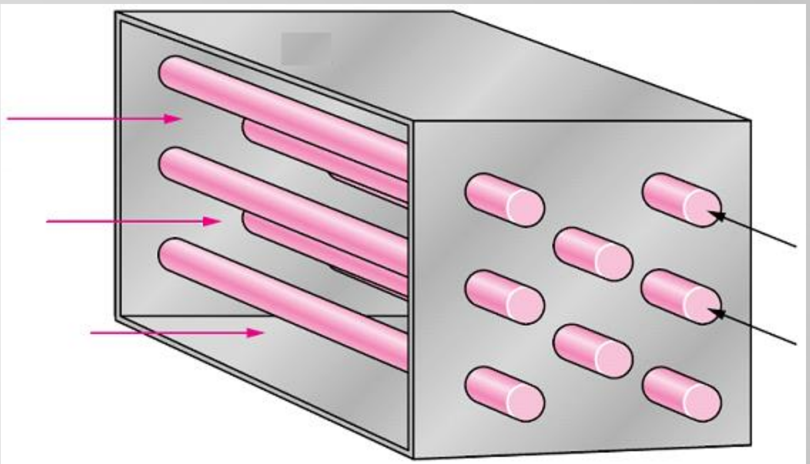
$$m = \rho V = \rho \tfrac{1}{6}\pi D^3 = (8055\ \text{kg/m}^3)\ \tfrac{1}{6}\pi(0.25\ \text{m})^3 = 65.9\ \text{kg}$$

$$Q_{\text{total}} = mC_p(T_2 - T_1) = (65.9\ \text{kg})(480\ \text{J/kg} \cdot {}^\circ\text{C})(300 - 200)^\circ\text{C} = 3{,}163{,}000\ \text{J}$$

$$\Delta t \approx \frac{Q}{\dot{Q}_{\text{ave}}} = \frac{3{,}163{,}000\ \text{J}}{610\ \text{J/s}} = 5185\ \text{s} = \mathbf{1\ h\ 26\ min}$$

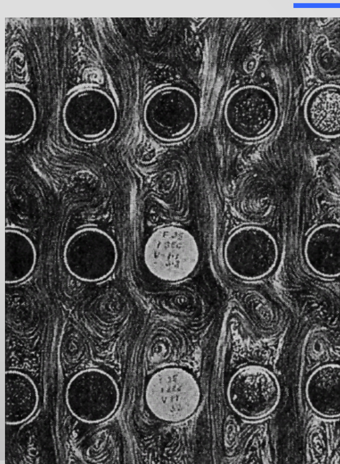
FLOW ACROSS TUBE BANKS

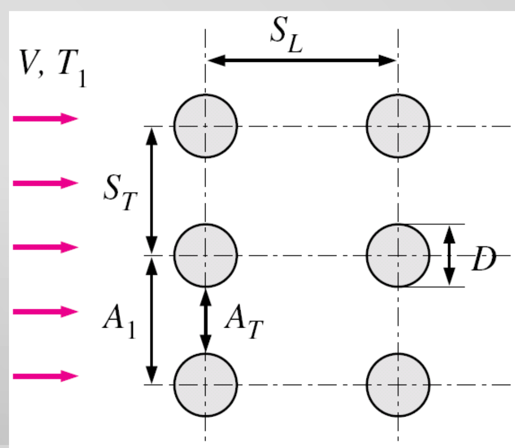
- Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such heat exchangers.
- In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.
- Flow through the tubes can be analyzed by considering flow through a single tube, and multiplying the results by the number of tubes.
- For flow over the tubes the tubes affect the flow pattern and turbulence level downstream, and thus heat transfer to or from them are altered.



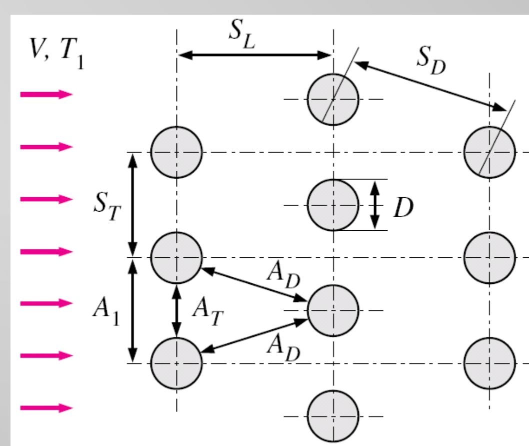
- Typical **arrangement (The tubes in a tube bank)**
  - in-line
  - staggered
- The **outer tube diameter** *D* is the **characteristic length**.
- The arrangement of the tubes are **characterized** by the
  - *transverse pitch* S_T,
  - *longitudinal pitch* S_L, and the
  - *diagonal pitch* S_D between tube centers.

In-line





Staggered



- As the fluid enters the tube bank, the **flow area decreases** from $A_1 = S_T L$ to $A_T = (S_T - D)L$ between the tubes, and thus flow velocity increases.
- In tube banks, the **flow characteristics** are dominated by the **maximum velocity** V_{max}.
- The Reynolds number is defined on the basis of **maximum velocity** as

$$\mathrm{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{V_{\max} D}{\nu}$$

- For ***in-line*** arrangement, the maximum velocity occurs at the **minimum flow area** between the tubes

$$V_{\max} = \frac{S_T}{S_T - D} V$$

- In *staggered* arrangement,
  - for $S_D > (S_T + D)/2$: $$V_{max} = \frac{S_T}{S_T - D} V$$
  - for $S_D < (S_T + D)/2$: $$V_{max} = \frac{S_T}{2(S_D - D)} V$$
- The nature of flow around a tube in the **first row resembles flow** over a **single tube**.
- The nature of flow around a tube in the **second and subsequent rows** is very **different**.
- The level of **turbulence**, and thus the heat transfer **coefficient**, **increases** with **row number**.
- there is **no significant change** in turbulence level **after the first few rows**, and thus the heat transfer **coefficient remains constant**.

- **Zukauskas** has proposed correlations whose general form is

$$Nu_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\left(\mathrm{Pr}/\mathrm{Pr}_s\right)^{0.25}$$

- where the values of the constants *C*, *m*, and *n* depend on **Reynolds** number.
- The **average Nusselt** number relations in Table 7–2 are for tube banks with **16 or more rows**.

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < \mathrm{Pr} < 500$ (from Zukauskas, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$\mathrm{Nu}_D = 0.9\,\mathrm{Re}_D^{0.4}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	100–1000	$\mathrm{Nu}_D = 0.52\,\mathrm{Re}_D^{0.5}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\mathrm{Nu}_D = 0.27\,\mathrm{Re}_D^{0.63}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\mathrm{Nu}_D = 0.033\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.4}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
Staggered	0–500	$\mathrm{Nu}_D = 1.04\,\mathrm{Re}_D^{0.4}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	500–1000	$\mathrm{Nu}_D = 0.71\,\mathrm{Re}_D^{0.5}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\mathrm{Nu}_D = 0.35(S_T/S_L)^{0.2}\,\mathrm{Re}_D^{0.6}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\mathrm{Nu}_D = 0.031(S_T/S_L)^{0.2}\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$

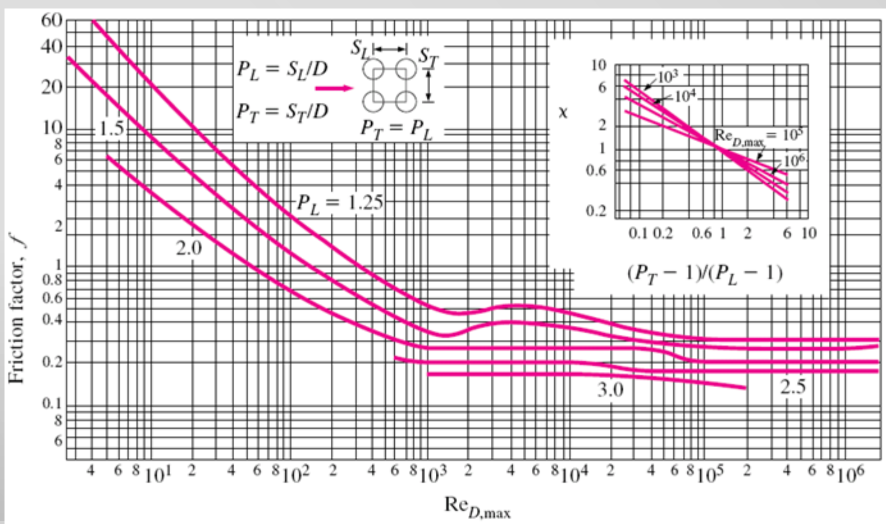
*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

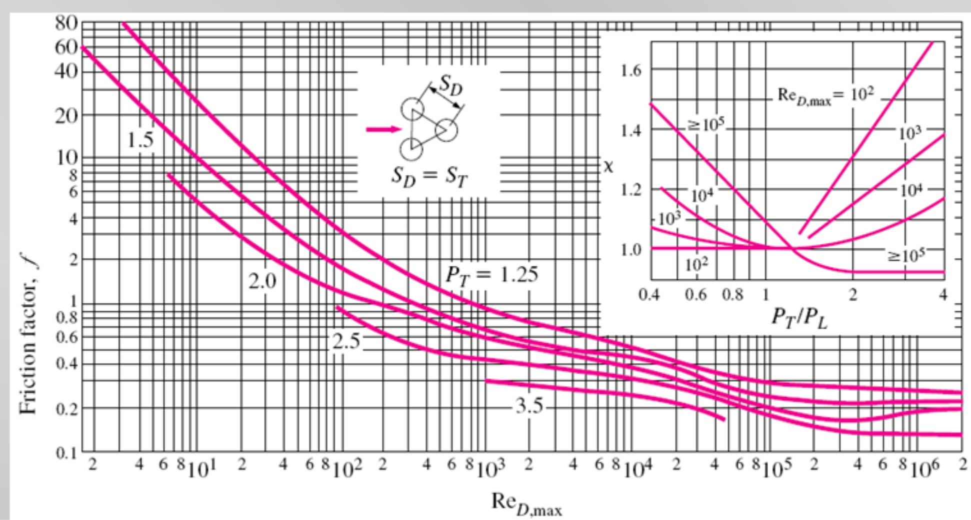
Pressure drop

- the ***pressure drop*** over tube banks is expressed as:

$$\Delta P = N_L f \chi \frac{\rho V_{max}^2}{2}$$

- f is the **friction factor** and χ is the **correction factor**.
- The **correction factor** (χ) given in the insert is used to account for the effects of **deviation** from **square** arrangement (in-line) and from **equilateral** arrangement (staggered).





- Those relations can also be used for tube banks with N_L provided that they are **modified** as

$$Nu_{D,N_L} = F \cdot Nu_D$$

- *The **correction factor***

  F values are given in

TABLE 7–3

Correction factor F to be used in $Nu_{D,N_L} = FNu_D$ for $N_L < 16$ and $Re_D > 1000$ (from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

- Logarithmic mean temperature difference (ΔT_{ln}) defined as

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

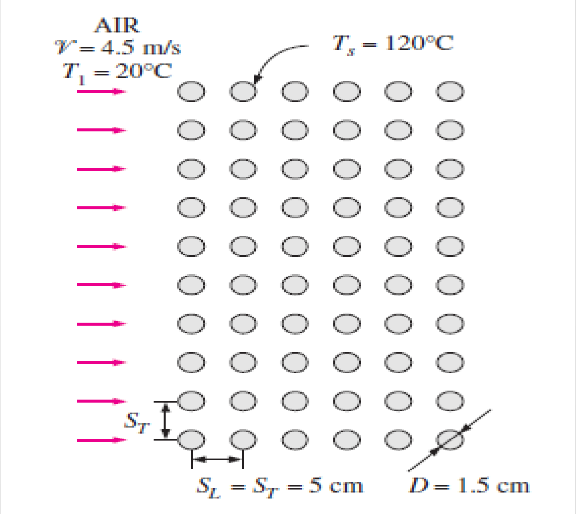
We will also show that the exit temperature of the fluid T_e can be determined from

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right)$$

- Where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho V(N_T S_T L)$ is the mass flow rate of the fluid.

Example Preheating Air by Geothermal Water in a Tube Bank

- In an industrial facility, air is to be preheated before entering a furnace by geothermal water at 120ºC flowing through the tubes of a tube bank located in a duct. Air enters the duct at 20ºC and 1 atm with a mean velocity of 4.5 m/s, and flows over the tubes in normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of *SL* = *ST* 5 cm. There are 6 rows in the flow direction with 10 tubes in each row, as shown in Figure below. Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank.



Assumptions 1 Steady operating conditions exist. 2 The surface temperature of the tubes is equal to the temperature of geothermal water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 60ºC (will be checked later) and 1 atm are Table A–15):

$$k = 0.02808 \text{ W/m} \cdot \text{K}, \qquad \rho = 1.06 \text{ kg/m}^3$$
$$C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \qquad \text{Pr} = 0.7202$$
$$\mu = 2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr}_s = \text{Pr}_{@T_s} = 0.7073$$

Also, the density of air at the inlet temperature of 20ºC (for use in the mass flow rate calculation at the inlet) is $\rho_1 = 1.204$ kg/m^3

Analysis It is given that $D = 0.015$ m, $S_L = S_T = 0.05$ m, and $\mathcal{V} = 4.5$ m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$\mathcal{V}_{\max} = \frac{S_T}{S_T - D}\mathcal{V} = \frac{0.05}{0.05 - 0.015}(4.5 \text{ m/s}) = 6.43 \text{ m/s}$$
$$\text{Re}_D = \frac{\rho \mathcal{V}_{\max} D}{\mu} = \frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})(0.015 \text{ m})}{2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 5091$$

The average Nusselt number is determined using the proper relation from Table 7–2 to be

$$\text{Nu}_D = 0.27 \text{ Re}_D^{0.63} \text{ Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$$
$$= 0.27(5091)^{0.63}(0.7202)^{0.36}(0.7202/0.7073)^{0.25} = 52.2$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case, the number of rows is $N_L = 6$, and the corresponding correction factor from Table 7–3 is $F = 0.945$. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,\,N_L} = F\text{Nu}_D = (0.945)(52.2) = 49.3$$

$$h = \frac{\text{Nu}_{D,\,N_L}k}{D} = \frac{49.3(0.02808 \text{ W/m}\cdot{}^\circ\text{C})}{0.015 \text{ m}} = 92.2 \text{ W/m}^2\cdot{}^\circ\text{C}$$

The total number of tubes is $N = N_L \times N_T = 6 \times 10 = 60$. For a unit tube length ($L = 1$ m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 60\pi(0.015 \text{ m})(1 \text{ m}) = 2.827 \text{ m}^2$$

$$\dot{m} = \dot{m}_1 = \rho_1 \mathcal{V}(N_T S_T L)$$
$$= (1.204 \text{ kg/m}^3)(4.5 \text{ m/s})(10)(0.05 \text{ m})(1 \text{ m}) = 2.709 \text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right)$$
$$= 120 - (120 - 20)\exp\left(-\frac{(2.827 \text{ m}^2)(92.2 \text{ W/m}^2\cdot{}^\circ\text{C})}{(2.709 \text{ kg/s})(1007 \text{ J/kg}\cdot{}^\circ\text{C})}\right) = 29.11^\circ\text{C}$$

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{(120 - 29.11) - (120 - 20)}{\ln[(120 - 29.11)/(120 - 20)]} = 95.4°C$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (92.2\ W/m^2 \cdot °C)(2.827\ m^2)(95.4°C) = \mathbf{2.49 \times 10^4\ W}$$

The rate of heat transfer can also be determined in a simpler way from

$$\dot{Q} = hA_s\Delta T_{\text{in}} = \dot{m}C_p(T_e - T_i)$$
$$= (2.709\ kg/s)(1007\ J/kg \cdot °C)(29.\ 11 - 20)°C = 2.49 \times 10^4\ W$$

For this square in-line tube bank, the friction coefficient corresponding to $Re_D = 5088$ and $S_L/D = 5/1.5 = 3.33$ is, from Fig. 7–27*a*, $f = 0.16$. Also, $\chi = 1$ for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f \chi \frac{\rho V_{max}^2}{2}$$
$$= 6(0.16)(1)\frac{(1.06\ kg/m^3)(6.43\ m/s)^3}{2}\left(\frac{1N}{1\ kg \cdot m/s^2}\right) = \mathbf{21\ Pa}$$

Pass Semester Quiz (June 2015)

Question 1

The South China Sea is a marginal sea that is part of the Pacific Ocean, encompassing an area from the Singapore and Malacca Straits to the Strait of Taiwan of around 3,500,000 square kilometers (1,400,000 sq mi). The area's importance largely results from one-third of the world's shipping transiting through its waters. The average water temperature of the Sungai Maong in August 2015 was recorded at 29°C. Consider a 30 mm outer diameter pipe is to cross Sungai Maong at 45 m wide section while being completely immersed in water. The average flow velocity of water is 3 m/s. Determine the total drag force in kN that is exerted on the pipe by the river.

Pass Semester Test (June 2015)

Question 1

An average person generates heat at a rate of 85 W while resting. Assuming one quarter of this heat is lost from the head and disregarding radiation, determine the average surface temperature of the head when it is not covered and is subjected to winds at 10°C and 35 km/h. The head can be approximated as a 30 cm diameter sphere. The flow of air is measured to be at 1 atm pressure and 15°C.

Pass Semester Exam (June 2015)

Question 1

Heat transfer through a liquid or gas can occur through conduction or convection depending on the presence of any bulk fluid motion.

i. Convection heat transfer depends on the fluid properties. State **THREE (3)** fluid properties of convection heat transfer.

ii. State **ONE (1)** similarity and difference of physical mechanism between heat conduction and heat convection.

Pass Semester Exam (June 2015)

Question 1

A 15 cm diameter and 30 cm high cylindrical bottle contains cold water at 6ºC. The bottle is placed in windy air at 27ºC. The water temperature is measured to be 14ºC after 50 minutes of cooling. At the average temperature, the properties of water are given in density = 999.7 kg/m^3 and specific heat = 4194 J/kg·ºC. Disregarding radiation effects and heat transfer from the top and bottom surfaces, calculate the convection heat transfer coefficient.

- END OF SLIDE -