TECHNIQUES OF DIFFERENTIATION

PRODUCT RULE

$$y = u.v$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

EXAMPLE

a

$$y = 3x\,(2x + 5)$$

$$u = 3x \qquad v = 2x + 5$$

$$\frac{du}{dx} = 3 \qquad \frac{dv}{dx} = 2$$

$$\frac{dy}{dx} = 3x\,(2) + (2x + 5)\,(3)$$

$$\frac{dy}{dx} = 6x + 6x + 15$$

$$\frac{dy}{dx} = 12x + 15$$

EXAMPLE

b

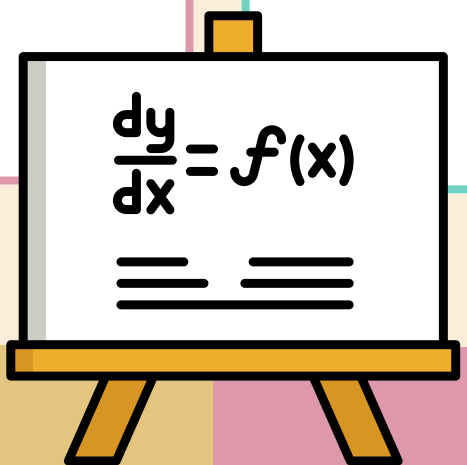
$$y = 2x^3e^{3x}$$

$$u = 2x^3 \qquad v = e^{3x}$$

$$\frac{du}{dx} = 6x^2 \qquad \frac{dv}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = 2x^3\left(3e^{3x}\right) + e^{3x}\left(6x^2\right)$$

$$\frac{dy}{dx} = 6e^{3x}x^3 + 6e^{3x}x^2$$



TECHNIQUES OF DIFFERENTIATION

QUOTIENT RULE

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

EXAMPLE

a

$$y = \frac{2x}{(3x+2)^4}$$

$u = 2x$ $\qquad$ $v = (3x+2)^4$

$\frac{du}{dx} = 2$ $\qquad$ $\frac{dv}{dx} = 4(3x+2)^3(3)$

$\frac{dv}{dx} = 12(3x+2)^3$

$$\frac{dy}{dx} = \frac{(3x+2)^4(2) - (2x)\left(12(3x+2)^3\right)}{\left((3x+2)^4\right)^2}$$

$$\frac{dy}{dx} = \frac{2(3x+2)^3((3x+2) - (12x))}{(3x+2)^8}$$

$$\frac{dy}{dx} = \frac{2(3x+2)^3(-9x+2)}{(3x+2)^8}$$

EXAMPLE

b

$$y = \frac{e^{(5x+2)}}{sin(4x+3)}$$

$u = e^{(5x+2)}$ $\qquad$ $v = sin(4x+3)$

$\frac{du}{dx} = 5e^{(5x+2)}$ $\qquad$ $\frac{dv}{dx} = 4cos(4x+3)$

$$\frac{dy}{dx} = \frac{(sin(4x+3)\left(5e^{(5x+2)}\right) - \left(e^{(5x+2)}\right)(4cos(4x+3))}{(sin(4x+3))^2}$$

$$\frac{dy}{dx} = \frac{\left(e^{5x+2}\right)((5sin(4x+3)) - (4cos(4x+3))}{sin^2(4x+3)}$$

TECHNIQUES OF DIFFERENTIATION

CHAIN RULE

$$y = f(u) \qquad u = g(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

EXAMPLE

a

$$y = ln\,(tan4x)$$

$$u = tan\,(8x) \qquad y = ln\,(u)$$

$$\frac{du}{dx} = 4sec^2\,(4x) \qquad \frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u} \times 4sec^2\,(4x)$$

$$\frac{dy}{dx} = \frac{4sec^2 4x}{tan4x}$$

EXAMPLE

b

$$y = \left(2x^4 - 1\right)^3$$

$$u = 2x^4 - 1 \qquad y = u^3$$

$$\frac{du}{dx} = 8x^3 \qquad \frac{dy}{dx} = 3u^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = 3u^2 \times 8x^3$$

$$\frac{dy}{dx} = 24u^2x^3$$

$$\frac{dy}{dx} = 24\left(2x^4 - 1\right)^2 x^3$$

LET`S DO EXERCISE

Find the derivatives of the following functions :

QUESTION

$a) y = 3x(2x+5)^4$

$b) y = x^3(7-4x)^2$

$c) y = (x^2+2)(8x-3)^4$

$d) y = x^7(7x-3)^2$

$e) y = \dfrac{5}{(3x+7)}$

$f) y = \dfrac{3x}{9x^2-1}$

$g) y = \dfrac{(4x+2)^3}{10x}$

$h) y = \dfrac{x-1}{(6x+5)^5}$

$i) y = \dfrac{cosx}{sin2x}$

$j) y = \dfrac{e^{7x+2}}{cos2x}$

ANSWER

$a) \dfrac{dy}{dx} = 15(2x+1)(2x+5)^3$

$b) \dfrac{dy}{dx} = 3x^2(7-4x)(7x-4x^2-8)$

$c) \dfrac{dy}{dx} = 2(24x^2-3x+32)(8x-3)^3$

$d) \dfrac{dy}{dx} = 7x^6(7z-3)(9x-3)$

$e) \dfrac{dy}{dx} = \dfrac{-15}{(3x+7)^2}$

$f) \dfrac{dy}{dx} = \dfrac{-27x^2}{(9x^2-1)^2}$

$g) \dfrac{dy}{dx} = \dfrac{4(2x+1)^2(4x-1)}{5x^2}$

$h) \dfrac{dy}{dx} = \dfrac{35-24x}{(6x+5)^6}$

$i) \dfrac{dy}{dx} = \dfrac{-sinxsin2x-2cosxcos2x}{(sin2x)^2}$

$j) \dfrac{dy}{dx} = \dfrac{e^{7x+2}(7cos2x+2sin2x)}{(cos2x)^2}$

LET`S DO EXERCISE

Find the derivatives of the following functions :

QUESTION

$a) y = (3x - 8)^5$

$a) y = \dfrac{2}{(4x+5)^3}$

$a) y = \dfrac{1}{\sqrt{3x^4 + 2}}$

$a) y = \sqrt{1 - e^x}$

$a) y = ln\sqrt{1 - 3x}$

$a) y = tan^6 x$

$b) y = sin^4 2x$

$c) y = cos^3\sqrt{2x - 1}$

$e) y = tan^{10} 3x^4$

$d) y = 7cos^5 (5x - 8)$

ANSWER

$a) \dfrac{dy}{dx} = 15(3x - 8)^4$

$a) \dfrac{dy}{dx} = \dfrac{-24}{(4x+5)^4}$

$a) \dfrac{dy}{dx} = \dfrac{1}{\sqrt{3x^4 + 2}}$

$a) \dfrac{dy}{dx} = \dfrac{-e^x}{2\sqrt{1 - e^x}}$

$a) \dfrac{dy}{dx} = \dfrac{-3}{2(1 - 3x)}$

$a) \dfrac{dy}{dx} = 6sec^2 x tan^5 x$

$b) \dfrac{dy}{dx} = 8cos2x sin^3 2x$

$c) \dfrac{dy}{dx} = \dfrac{-3sin\sqrt{2x-1}cos\sqrt{2x-1}}{\sqrt{2x-1}}$

$e) \dfrac{dy}{dx} = 120x^3 sec^2 3x^4 tan^9 3x^4$

$d) \dfrac{dy}{dx} = -175sin(5x - 8)cos^4 (5x - 8)$